NUCLEAR TRAINING COURSE

COURSE 127

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Nuclear Theory - Course 127

NUCLEAR STRUCTURE

The subjects discussed in this course cover many of the topics already discussed in the levels 4, 3 and 2 Nuclear Theory courses. Therefore, it is inevitable that this level 1 Nuclear Theory course will review much of the material already covered. However, the approach to the subject matter will be somewhat different and certainly more detailed. New topics will also be introduced here and there, so that when you have completed this course you should have a good basic knowledge of the physical processes occurring in nuclear reactors.

For the sake of convenience, this course will be divided into two parts. These are:

(a) Reactor Theory - The Steady State. This section deals with such topics as the critical size of a reactor, neutron balance in a reactor, moderator and reflector properties. All these discussions centre around a reactor in which the chain reaction is just being maintained and where no variations with time are occurring.

(b) Reactor Theory - Time Dependent Effects. This section could be considered as being a discussion of the effects that arise from a "Disturbance of the Steady State". The type of topics that would be covered in this section would be temperature change effects, effects of changes in reactivity, reactor regulation and protection, build-up of poison and other effects that vary over short or long periods of time.

The first four lessons of this course will deal with those aspects of nuclear physics which are important for an understanding of reactor theory.

Equivalence of Mass and Energy

Einstein showed that mass and energy are equivalent. The relationship between mass and energy changes may be written:

$$\Delta E = \Delta m c^2$$

Where ΔE is the energy change expressed in joules, Δm is the accompanying change in mass given in kilograms and c is the velocity of light, equal to 3×10^8 meters per second.

Δm mass in kg.
c vel. of light
ΔE joules.

A convenient and very common unit of energy in nuclear physics is the *electron volt* (abbreviated *eV*). It is the energy gained by an electron in being accelerated through a potential difference of 1 volt.

$$1 \text{ eV} = 1.6021 \times 10^{-19} \text{ joule}$$

$$1 \text{ keV} = 10^{3} \text{ eV}$$

$$1 \text{ MeV} = 10^{6} \text{ eV}$$

$^{A}_{Z}X$

A - mass no.

Z - atomic no.

The Nucleus, Nuclear Particles

The atomic nucleus consists of Z protons and N neutrons, where Z and N are the *atomic number* and *neutron number* respectively. The total number of *nucleons* in the nucleus, that is, neutrons and protons, is equal to $Z + N = A$, where A is the *atomic mass number*.

A nuclear species with a given Z and a given A is called a *nuclide*. To distinguish a particular nuclide it is written in the form $_{Z}X^{A}$, where X is the chemical symbol for the element. Nuclides with the same Z but different A are called *isotopes*. Every element has a number of isotopes - stable and unstable - which range from 3 (hydrogen) to 26 (tin), with an average of about 10 isotopes per element.

The mass of the proton is 1.67252×10^{-27} kg. It carries a positive charge of 1.60210×10^{-19} coulombs (C), equal in magnitude to the negative charge of the electron, and it is a stable particle.

The mass of the neutron is marginally greater than that of the proton, namely 1.67482×10^{-27} kg, and it is electrically neutral. The neutron is not stable unless it is bound in a nucleus. A free neutron decays to a proton with the emission of a β^- particle and an antineutrino, a process which on the average occurs in about 12 minutes. You will see later in this course that the average lifetime of neutrons in a reactor before they are absorbed or leak from the system is no greater than a millisecond. The instability of the neutron is therefore of no consequence in reactor theory.

$_{0}n^{1} \longrightarrow {}_{1}p^{1} + {}_{-1}e^{0} + \bar{\nu}$ antineutrino.

$_{1}p^{1} \longrightarrow {}_{0}n^{1} + {}_{-1}e^{0} + \nu$ neutrino.

Nuclear Masses

The masses of atoms are conveniently expressed in *atomic mass units*, or *amu*. The actual mass of a nucleus is measured on a scale, the *physical scale*, such that the mass of the neutral C^{12} atom is precisely 12 amu, and hence 1 amu = 1.660438×10^{-27} kg.

TABLE 1 - SOME USEFUL NUMBERS

Avogadro's Number	N_o	6.02252×10^{26} (kg.mole)$^{-1}$
Electron rest mass	m_e	9.1091×10^{-31} kg 5.48597×10^{-4} amu 0.511006 MeV
Elementary charge	e	1.60210×10^{-19} coul
Neutron rest mass	m_n	1.67482×10^{-27} kg 1.0086654 amu 939.550 MeV
Proton rest mass	m_p	1.67252×10^{-27} kg 1.0072766 amu 938.256 MeV
Speed of light	c	2.997925×10^{8} m.s^{-1}

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1 MeV	10^6 eV 1.60210×10^{-13} joule
1 amu	1.660438×10^{-27} kg 931.478 MeV 1.49232×10^{-10} joule
1 watt	1 joule/sec $\equiv 3.1 \times 10^{10}$ fissions/sec
1 day	86400 sec
1 year	3.156×10^7 sec
1 curie	3.70×10^{10} dps

The atomic mass of a nuclide should be distinguished from the chemical atomic weight which is the average weight of a large number of atoms of a given element. It is not quite the same as the mass of an individual atom unless the element contains a single isotope. Furthermore, you should note that the atomic weight unit on the *chemical scale* is defined as one-sixteenth of the average weight of an oxygen atom in a natural mixture of stable oxygen isotopes (0.204% O^{18}, 0.037% O^{17} and the rest O^{16}). In many calculations this slight distinction (about 3 ppm) is insignificant and the atomic mass, denoted by *A*, is used rather loosely.

Binding Energy

The mass of the proton is 1.00728 amu, and the mass of the neutron is 1.00867 amu. The actual mass of a nuclide is not equal to the total mass of its individual nucleons, the difference being called the *mass defect*. This mass defect is a consequence of the equivalence of mass and energy and arises from the *binding energy* of the nuclide. This is the energy required to split the nuclide into its individual component nucleons. From experiment it is found that, except for a few light nuclides, the binding energy per nucleon in the nucleus increases rapidly as the size of the nucleus increases up to about A = 60, but for greater values it decreases again gradually. This means that nuclei of intermediate mass are more strongly bound than the light and the heavy nuclei. Thus energy may be released by combining two light nuclei (*fusion*) or by splitting a heavy one into two of intermediate mass (*fission*).

Nuclear Forces

Between two electric charges of the same sign there is a repulsive force which is called a *Coulomb force*. Since nuclei may contain a large number of positive protons each repelling the others due to Coulomb forces it is clear that there must be other forces present which are attractive. These are short range *nuclear forces*. They act between all adjacent nucleons, whether n-p, n-n or p-p, and drop off rapidly on separation of the nucleons.

The lighter stable nuclei contain roughly equal numbers of neutrons and protons (eg, ${}_6C^{12}$, ${}_8O^{16}$, ${}_9F^{19}$, ${}_{11}Na^{23}$). As the number of protons in the nucleus increases, the long range Coulomb forces build up more rapidly than the nuclear forces which only have short range. Therefore, in order for heavier nuclei to remain intact more neutrons are required to supply binding forces between all particles to overcome the distruptive Coulomb forces. As a result, the n/p ratio gradually increases from 1 (light nuclei) to about 1½ (eg, ${}_{82}Pb^{208}$, ${}_{90}Th^{232}$, ${}_{92}U^{238}$). For reasons we will not go into here, there is a limit to the number

n/p ratio

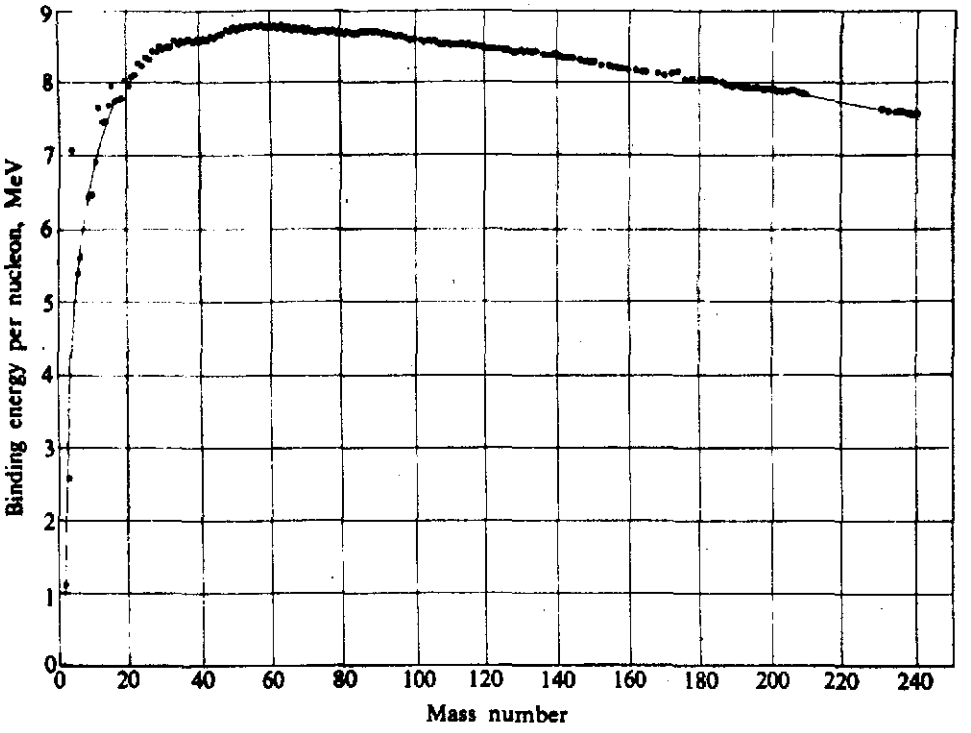


Fig. 1 Binding Energy vs Mass Number

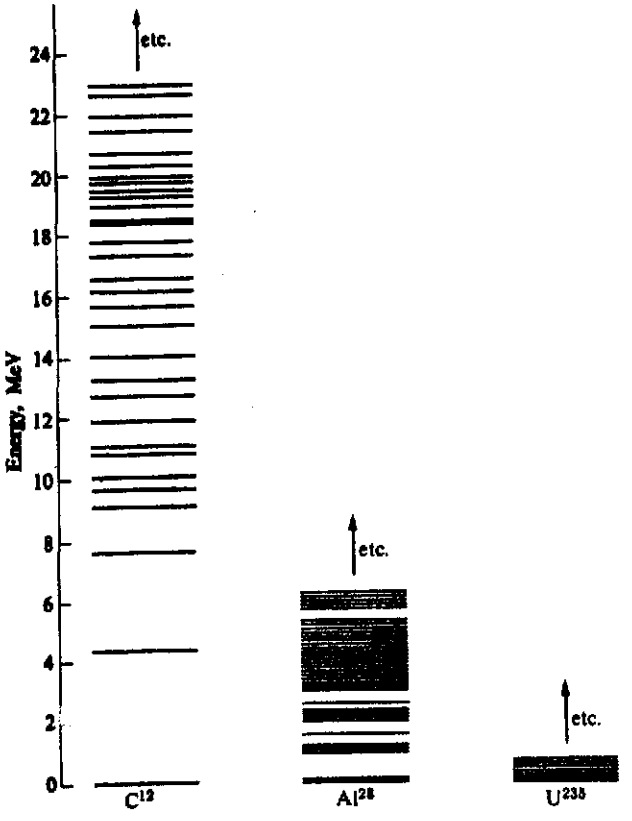


Fig. 2 Nuclear Energy Levels of C^{12}, Al^{28} and U^{235}

of excess neutrons a nucleus can live with, and as a result the heavy nuclei are all unstable and there are no naturally occurring elements having a value of *A* greater than 238.

Nuclear Energy Levels

A nucleus is said to be in its *ground state* when the nucleons are arranged in such a way that the potential energy is a minimum. If it is not in its ground state it is said to be in an *excited state* and the excess of energy is called *excitation energy*. The potential energy does not take on a continuous range of values, but has discrete values which are termed *energy levels*. For heavy nuclei these energy levels have a minimum separation of about 0.1 MeV, for light nuclei this separation is much greater.

Radioactivity

All the naturally occurring nuclides heavier than lead ($Z = 82$) and a few lighter nuclides are unstable and are *naturally radioactive*. They decay by emitting either an *alpha particle* (helium nucleus) or a *beta particle* (fast electron). In most cases the resulting nucleus, or *daughter*, is produced in an excited state. It then decays to its ground state by the emission of one or more *gamma photons*. Usually, but not always, this occurs instantaneously, ie, within 10^{-14} seconds of the formation of the daughter. A radioactive nuclide, *or radionuclide*, may also decay by capturing one of the inner orbital electrons and this is known as *K capture*. The γ photons after leaving the nucleus may be absorbed by ejecting an electron from an orbit of the same atom; this gives rise to secondary β particles, the process being known as *internal conversion*.

Radioactivity is governed by only one fundamental law, namely that the probability of a radionuclide decaying in unit time is constant and independent of external conditions. This constant is called the *decay constant* and is denoted by λ.

Consider a sample of radioactive material containing only one kind of radionuclide. If there are n atoms of this nuclide present at time t, then the number decaying in the time interval between t and t + dt is $\lambda n\, dt$. The number of atoms is therefore reduced by dn, where:

$$dn = -\lambda n\, dt$$

or

$$n = n_0 e^{-\lambda t}$$

where n_0 is the number of atoms at time $t = 0$.

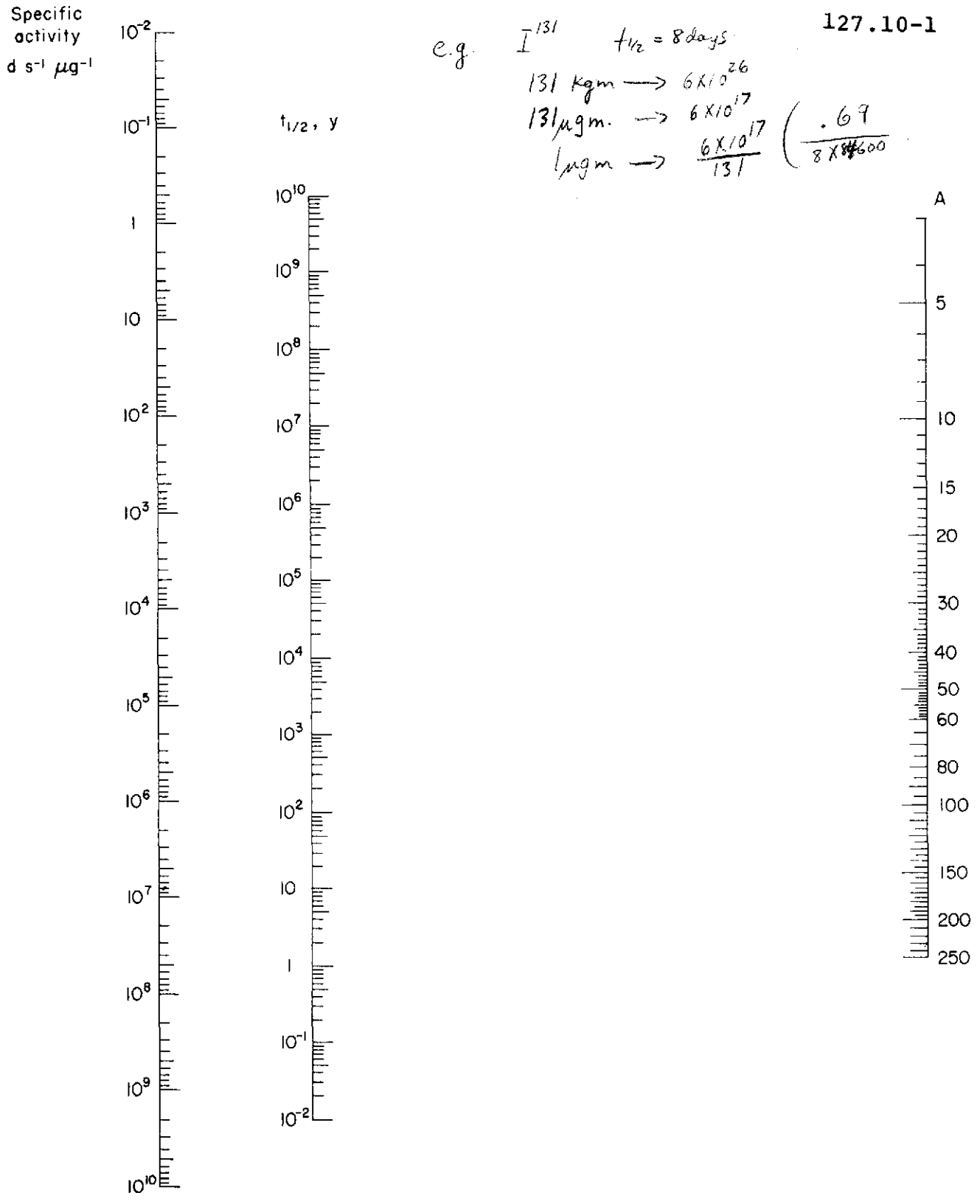


Fig. 3 Nomogram for Calculating Specific Activities

Place a straight edge over the values of the mass number (A) and the half life (in years), and read where it crosses the scale for specific activity in disintegrations per second per microgram ($ds^{-1}\ \mu g^{-1}$)

The time taken for the number of atoms to be diminished to one half is called the *half-life*, and is denoted by $t_{\frac{1}{2}}$. It is not hard to show that:

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The *activity* of the sample is just the number of disintegrations per second, $\frac{dn}{dt}$, ie, λn. The unit of activity is the *curie* (abbreviated *Ci*), which corresponds to a decay rate of 3.70×10^{10} disintegrations per second.

Before we continue with an example, it would probably be best to refresh your memory with respect to *Avogadro's number*, N_o. This is the number of molecules per kg.mole, or the number of atoms per kg.atom. A kg.atom is *A* kg where *A* is the atomic mass of the substance (in amu). $N_o = 6.0225 \times 10^{26}$ $(\text{kg.mole})^{-1}$ and this number holds for all substances. For example, 131 kg of I^{131} contain N_o I^{131} atoms and 18 kg of water also contain N_o water molecules.

Example: Calculate the activity due to K^{40} in an 80 kg man, assuming that 0.35% of the body weight is potassium. The natural abundance of K^{40} in potassium is 0.0118%, and $t_{\frac{1}{2}} = 1.27 \times 10^9$ years.

$$\text{Mass of } K^{40} = 80 \times 0.35 \times 10^{-2} \times 0.0118 \times 10^{-2}$$

$$= 33 \times 10^{-6}$$

The number of K^{40} atoms, n, is then given by:

$$n = 33 \times 10^{-6} \times \frac{6 \times 10^{26}}{40}$$

$$= 5 \times 10^{20}$$

The activity now becomes:

$$\lambda n = \frac{0.69}{1.27 \times 10^9} \times 5 \times 10^{20} \text{ disintegrations per year}$$

$$= 8000 \text{ dps}$$

$$= \frac{8000}{3.7 \times 10^4} \ \mu\text{Ci}$$

$$= 0.22 \ \mu\text{Ci}$$

The five pages following this lesson show a *Chart of the Nuclides*. This is a plot of all known nuclides and some of their properties, such as direction of decay, half life and natural abundance. There are charts with considerably more detail, usually much bigger and printed in multi colors, which you can refer to in the stations. The chart reproduced here (by kind permission of Encyclopaedia Brittanica) nevertheless gives you most of the information you would normally need to look up. In fact, you will have to refer to it in order to do some of the assignments below.

ASSIGNMENTS

1. Assuming all the mass of an atom to be concentrated in the nucleus, calculate the density of the nucleus in tonnes (1 Mg) per cubic millimeter. The radius of a nucleus of mass number A is believed to be approximately given by $r = 1.2 \times 10^{-15} A^{1/3}$ m.

2. A carefully purified sample of U^{238} weighs 6.70 mg and undergoes 85 dps. What is the half-life of U^{238}?

3. How much helium at STP will be formed from 1 g U^{238} in one million years?

4. The ~~quantity~~ of radiographs taken with x or γ rays is improved as the size of the radiation source is decreased. On this basis, compare the merits of 5 Ci Ra^{226} and 5 Ci Rn^{222} at atmospheric pressure. Consider each source to be spherical.

5. Show that the mean lifetime of radioactive atoms is about 1.4 times their half-life.

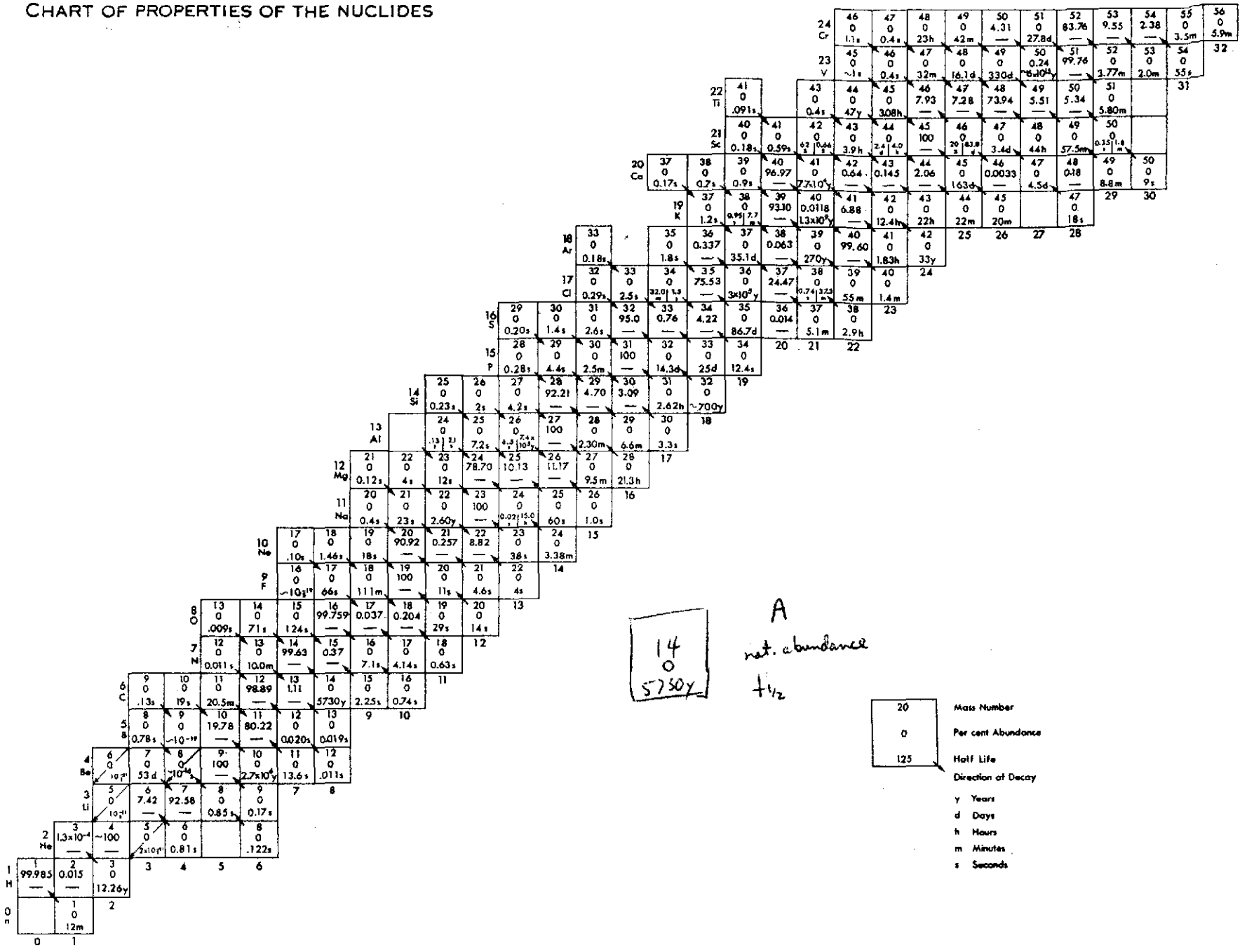
6. U^{235} has a half-life of 1.2×10^{17} years for spontaneous fission. Estimate the rate of spontaneous fissioning for 1 gram of U^{235}.

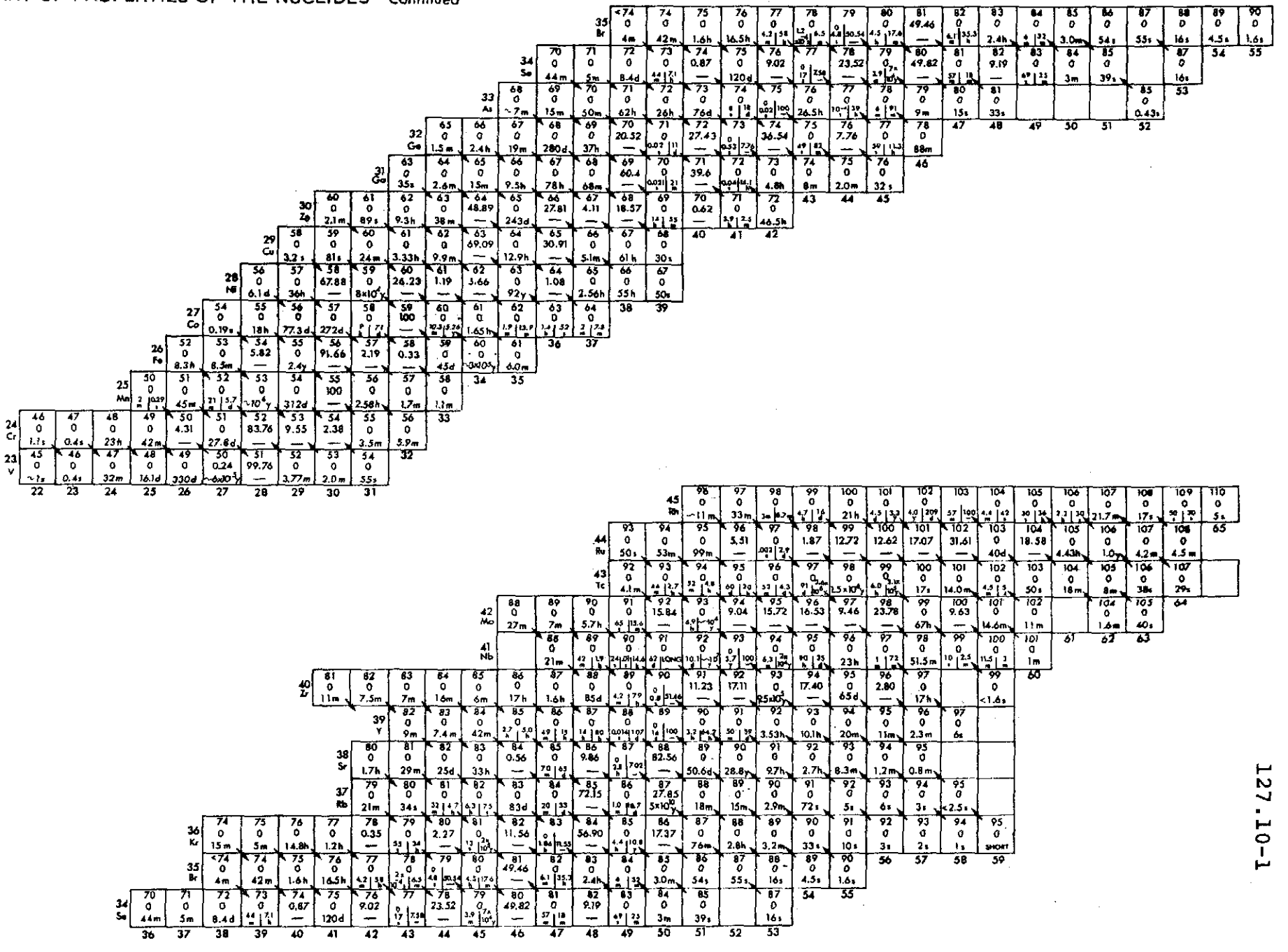
7. As a result of fuel failure, I^{131} activity in the delay tank is 6 Ci. The tank has a volume of 0.85 m^3 and the half-life of I^{131} is 8 days. The permissible release rate through the ventilation exhaust system is 0.05 μCi/s. Calculate the total weight of I^{131} in the tank and how long it must be held there before the tank can be exhausted at a rate of 2.4×10^{-4} $m^3.s^{-1}$.

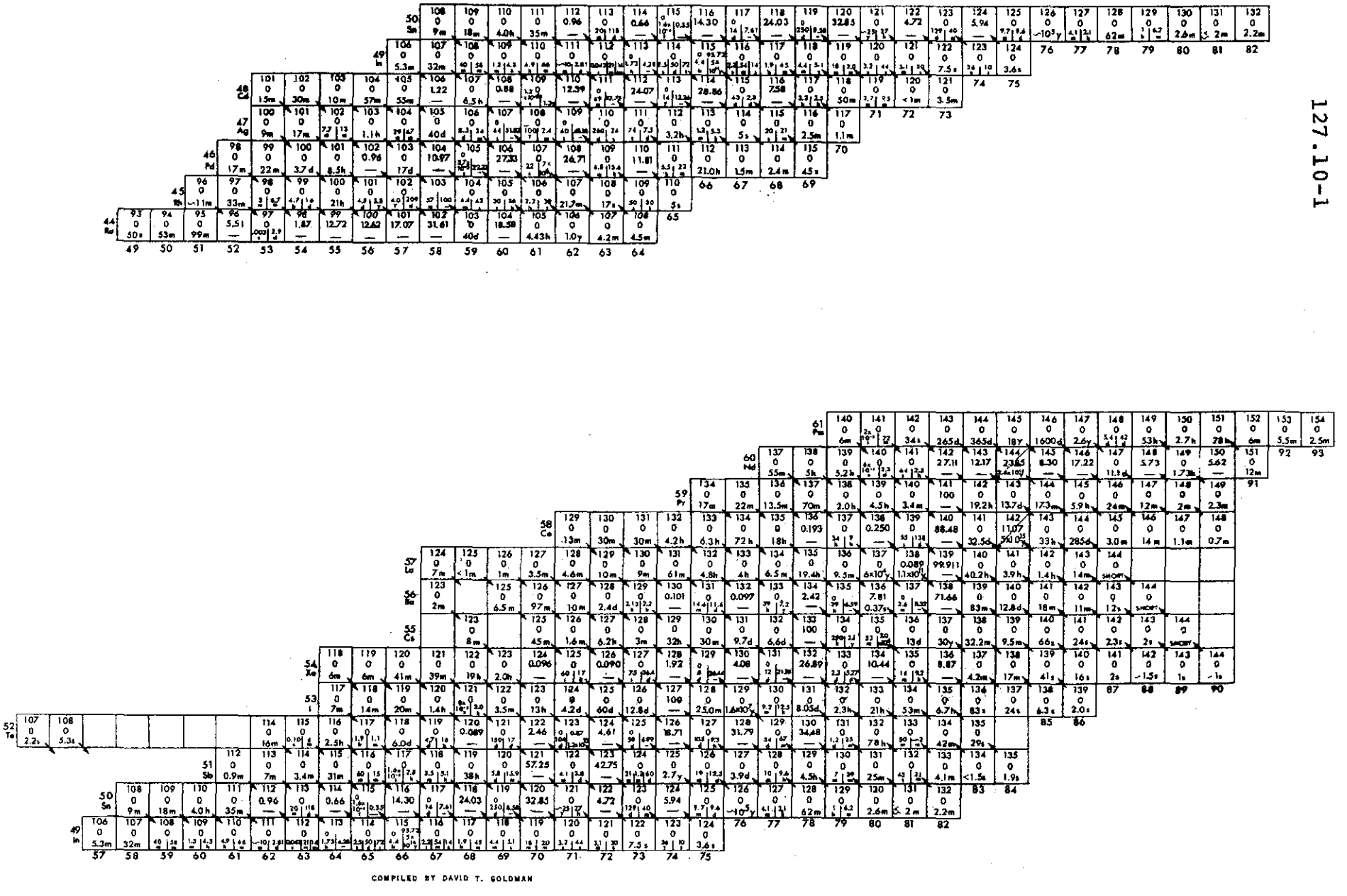
J.U. Burnham

$_Z X^A$ isobar isotope

CHART OF PROPERTIES OF THE NUCLIDES







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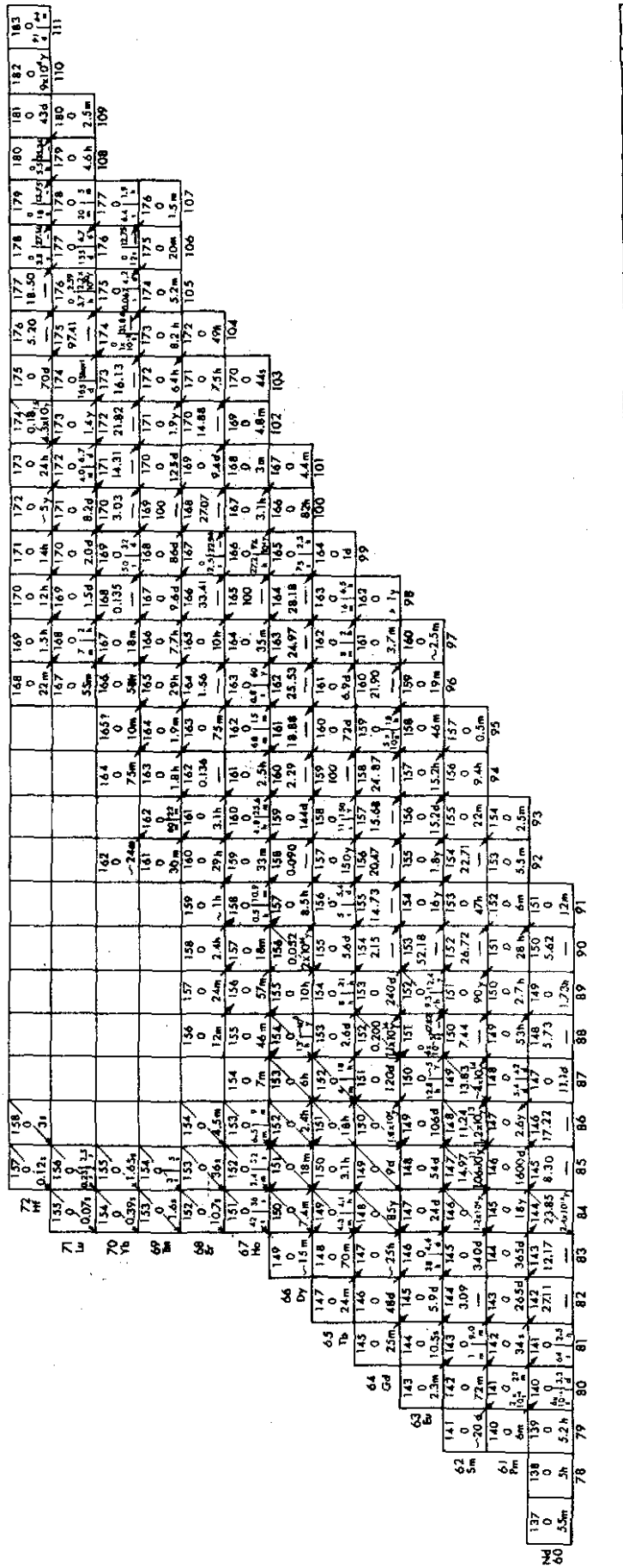

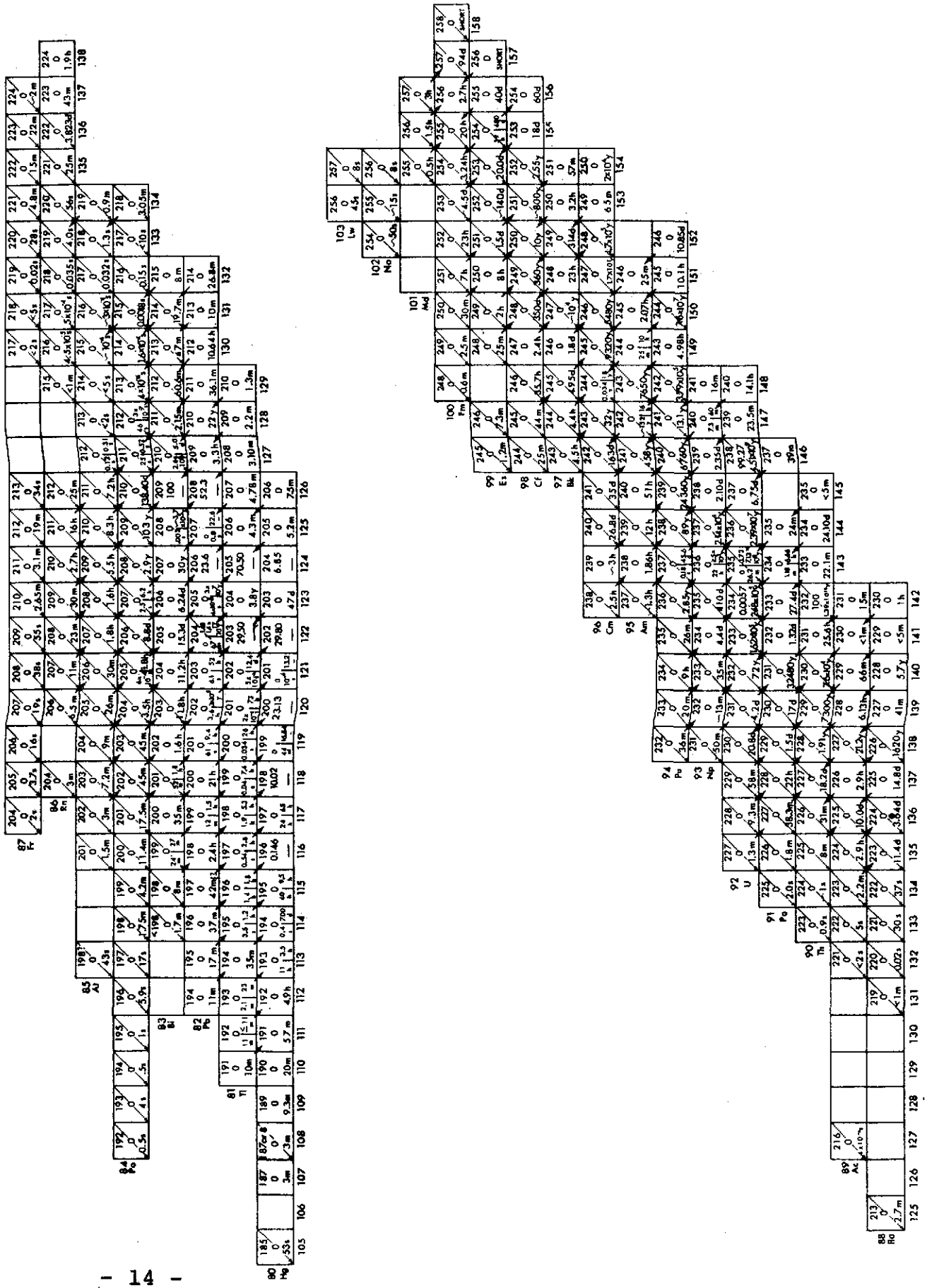

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NEUTRON REACTIONS

Nuclear reactions can occur as a result of collisions between various particles or gamma photons and nuclei. Nuclear charges particles, such as protons, deuterons (deuterium or H^2) and alpha particles, need to have a large amount of energy (tens of MeV) before they are able to overcome the Coulomb repulsive forces and enter a nucleus.

Neutrons and gamma photons, however, are not charged and are therefore able to interact with nuclei very effectively, even when they have very little energy. In fact, generally speaking, there is a greater chance of a reaction occurring with low rather than high energy neutrons, because the former are in contact with the nucleus for a greater length of time.

The operation of a reactor basically depends on how neutrons react with nuclei in the reactor. It is therefore necessary to look at these reactions, called *neutron reactions*, in some detail. Although there are well over a dozen known neutron reactions, we need only consider the five that are of importance to us.

All neutron reactions can be categorized as either *elastic* or *inelastic collisions*, depending on whether kinetic energy is conserved in the collision or not.

Elastic Collisions

Elastic collisions are those in which the total kinetic energy before the collision is equal to that after the collision.

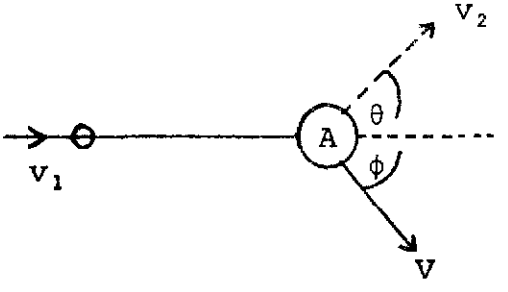

Fig. 1. Neutron Making an Elastic Collision

For example, consider a neutron of mass 1 amu and velocity v, striking a stationary nucleus of mass A amu, and bouncing off with velocity v_2. Since kinetic energy is conserved, we can say that

$$v_1^2 = v_2^2 + AV^2$$

You can see that some of the neutron's energy has been transferred to the target nucleus, the net result being a slowing down of the neutron. In fact, you should be able to show (from conservation of momentum) that the energy lost by the neutron in the collision, ΔE, is given by

$$\frac{\Delta E}{E} = \frac{4A \cos^2\phi}{(A+1)^2}$$

The importance of this expression will become evident later on when we discuss moderators and their properties.

The neutron is said to be *scattered* in the collision process, because the angle θ at which it bounces off the target nucleus depends on the angle at which it strikes it, and obviously this is quite random. Such reactions are therefore described by the term *elastic scattering*.

Inelastic Collisions

The neutron may enter the nucleus to form a *compound nucleus*. This is known as an inelastic collision, because kinetic energy is not conserved. Instead some of the neutron's kinetic energy is transformed into internal energy of the compound nucleus. The compound nucleus has too much energy to exist for any great length of time (no more than 10^{-14}s), and the reaction that then follows will be one of a number of possible alternatives.

1. The compound nucleus may get rid of its excess energy by emitting a neutron and a gamma photon. An example of this is shown in Fig. 2 on the following page. A neutron is shown entering a U-238 nucleus to form a U-239 nucleus. This emits a neutron (any one) and a gamma photon to become U-238 again. The net result again is a slowing down of the neutron, because the net energy it has lost has been transferred to the gamma photon. This reaction is known as *inelastic scattering*, because the direction of the emitted neutron is quite arbitrary. The reaction cannot occur unless the compound nucleus has gained sufficient energy to be raised to an excited state, ie, 0.1 MeV for heavy nuclei and much more for lighter ones. From a reactor physics point of view, we can probably ignore inelastic scattering other than with uranium atoms in the fuel where the neutron energies will be greatest.

elastic - same on (bounces)
inelastic - diff. on (absorbed + then re-emission).

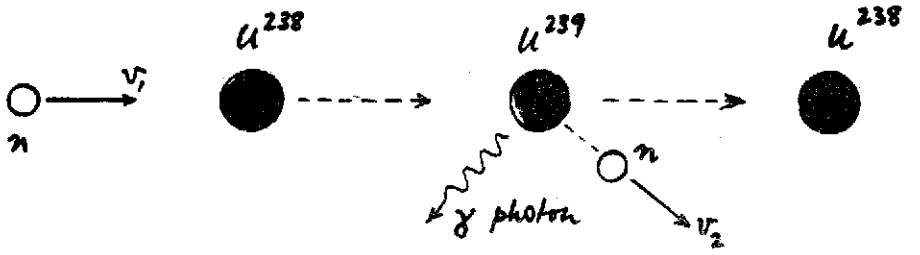

Fig. 2. Inelastic Scattering (n,n^1)

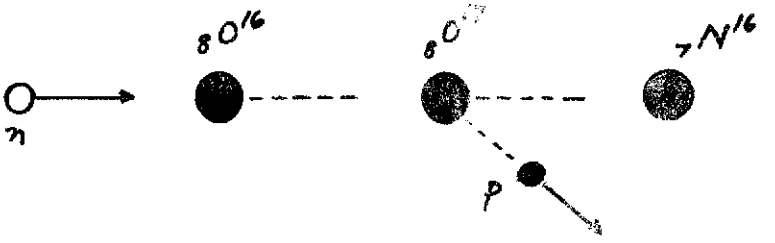

Fig. 3. Transmutation (n,p)

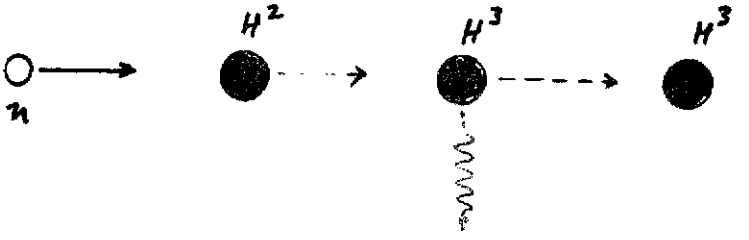

Fig. 4. Radiative Capture (n,γ)

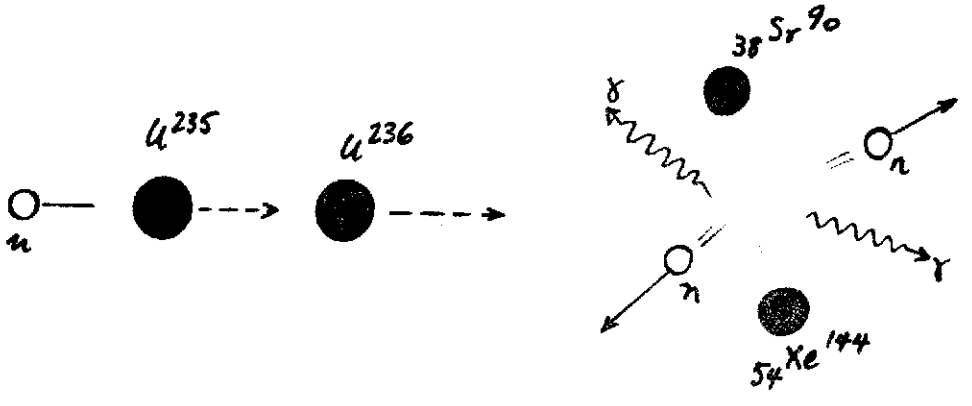

Fig. 5. Fission (n,f)

2. The compound nucleus may emit a charged particle (either a proton or an alpha particle) and so form an entirely new element. Fig. 3 shows such a *transmutation* of oxygen-16, which can occur if the neutron energy is greater than about 10 MeV. This reaction may be written as

$$_0n^1 + {}_8O^{16} \longrightarrow {}_7N^{16} + {}_1p^1$$

or you may prefer the short-hand version $O^{16}(n,p)N^{16}$. The N-16 is radioactive and emits high energy gamma radiation. It represents a radiation hazard in any region where O-16 (usually in water - H_2O or D_2O) is irradiated with high energy neutrons. The same applies to $O^{17}(n,p)N^{17}$.

There are a number of other (n,p) or (n,α) reactions of interest to us:-

$S^{32}(n,p)P^{32}$:- radiation monitoring film badges often contain a sulphur pellet. After exposure an estimate of the high energy (>2 MeV) neutron dose may be obtained by counting the activity of the phosphorus-32 (a β^- emitter with a 15 day half-life).

$B^{10}(n,\alpha)Li^7$:- reactor instrumentation for monitoring the neutron population utilises this reaction. It is possible at all neutron energies, and releases 2.5 MeV as kinetic energy of the helium and lithium nuclei. This can be detected relatively easily, even in the high gamma background of a reactor environment.

$He^3(n,p)H^3$:- very sensitive reactor instrumentation would make use of this reaction, because it occurs with much greater frequency than the above. He-3 counters were used for the initial start-up of the Pickering reactors.

3. The most common neutron reaction is *radiative capture*, so called because the compound nucleus has *captured* a neutron to *radiate* a gamma photon. Such gamma photons are frequently called *capture gammas*. Radiative capture can occur with practically all nuclei for all neutron energies. Generally speaking, its probability increases as the neutron energy decreases.

An example of such a reaction is shown in Fig. 4, which explains how tritium (hydrogen-3) is produced in heavy water reactors.

Radiative capture is important for two reasons:

(a) Any neutron capture in the reactor materials is undesirable because, simply speaking, it represents a waste of neutrons. The one exception to this is that the $U^{238}(n,\gamma)U^{239}$ reaction ultimately produces plutonium-239, which has desirable fuel properties.

(b) The product nucleus formed more times than not is radioactive and might present a radiation hazard. For example, corrosion products circulated by the heat transport system will be activated as they pass through the reactor core. When they later plate out in this system, the whole system becomes a radioactive hazard, and will remain so even if the reactor is shut down (ie, if the neutron source is removed). The three most troublesome activation products in our reactors are cobalt-60, manganese-56 and copper-64, and they are produced in this way.

Co^{60}
Mn^{56}
Cu^{64}

4. The final reaction we are going to consider is called *fission*. The word is borrowed from the biologists, who use it to describe the breaking up of a cell into two new ones. In the fission reaction the compound nucleus will usually split into two new nuclei (called *fission fragments*) and an average of two or three new neutrons. Generally speaking, fission reactions are relatively rare; an example is shown in Fig. 5. A detailed explanation of the fission process will be given in the next lesson.

ASSIGNMENT

1. (a) Why are neutrons more effective than charged particles in causing nuclear reactions?

   (b) Why are slow neutrons more likely to cause a nuclear reaction than fast neutrons?

2. Calculate the percentage energy loss of neutrons striking U-238 and H-2 nuclei in head-on and 45^0 collisions.

3. With the help of information given in lesson 127.10-1 explain why the energy required for inelastic scattering of neutrons has to be ~0.1 MeV for uranium, and much more than this for lighter nuclides.

4. What is the distinction between radiative capture and activation? Give examples.

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FISSION

After having looked at neutron reactions in general, we shall use this lesson to describe the fission reaction and its products in some detail.

The Fission Reaction

Production of nuclear power relies on the fact that some nuclei will fission, and that energy is released during this fission process because a loss of mass occurs ($E = mc^2$). There are two types of fission; *spontaneous* and *induced*.

(a) Spontaneous Fission

In this reaction, a nucleus fissions entirely spontaneously, without any apparent external cause. It is quite a rare reaction, generally only possible for nuclei with atomic masses of around 232 amu or more. (As the atomic mass number increases, spontaneous fission becomes more and more probable. One could argue that there is an infinite number of heavy elements which do not exist, because they are not stable against spontaneous fission decay.) The table below shows the spontaneous fission and alpha decay rates of the U-235 and U-238 isotopes.

TABLE 1

Spontaneous Fission and Alpha Decay Rates of Uranium

	$t_{\frac{1}{2}}(\alpha)$ (years)	$t_{\frac{1}{2}}$(s.f.) (years)	α decay rate (atoms $s^{-1}g^{-1}$)	s.f. decay rate (atoms $s^{-1}g^{-1}$)
U-235	7.1×10^{8}	1.2×10^{17}	79×10^{3}	0.3×10^{-3}
U-238	4.5×10^{9}	5.5×10^{15}	12×10^{3}	6.9×10^{-3}

From this table you will be able to appreciate that spontaneous fission has no significance in the production of power. Nevertheless, it is important in that it represents a small uncontrollable source of neutrons in a reactor.

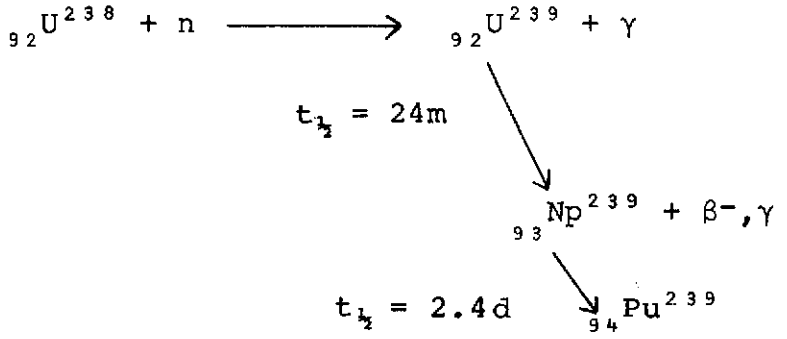
(b) Induced Fission

Certain heavy nuclei can be *induced* to fission as a result of neutron capture. In most cases the energy of the captured neutron must be very high before fission can occur, and therefore we can restrict our discussion to those nuclei which can be fissioned by neutron energies likely to be found in a reactor. In practice, we are then dealing with neutrons ranging from 10 MeV down to *thermal energies.**

Practical Fission Fuels

The only nuclei of practical importance to us are the U-233, U-235 and U-238 isotopes of uranium, and the Pu-239 and Pu-241 isotopes of plutonium. For all of these, except U-238, fission with thermal neutrons (*thermal fissions*) is much more probable than fission with fast neutrons (*fast fissions*). This is an important (and desirable) nuclear property, and such nuclides are said to be *fissile*. U-238, which will not fission with thermal neutrons, but which will fission with fast neutrons of energy greater than about 1.2 MeV, is merely said to be *fissionable*. It does, nevertheless, make a small contribution to the power produced in a reactor.

Natural uranium only contains U-235 (0.72%) and U-238. Over a period of reactor operation, Pu-239 and also some Pu-241 will be built up in the fuel as a result of neutron capture:



Pu-239 is fissile like U-235. If it does not undergo fission, it may capture a neutron to form Pu-240. Although this is fissionable it is much more likely to capture another neutron to form fissile Pu-241. A significant fraction of the total power produced by fuel during its life in our reactors is due to fission

* Thermal energy is the energy of a free neutron which is in thermal equilibrium with the atoms or molecules surrounding it. Neutrons of this energy are known as *thermal neutrons*. At 20°C, thermal energy is 0.025 eV.

of the fissile plutonium isotopes. We will deal with this in more detail later on in the course.

U-233 does not exist in natural uranium. This is a pity, because it has the most desirable properties of all of the fissile nuclides. It is of importance in reactor systems which convert thorium (Th-232) to U-233 by neutron capture followed by two successive β^- decays.

Fission Fragments

The fission fragments formed when spontaneous or induced fission occurs are two new nuclei. These may be any two of about 300 nuclides which are known to be formed as a result of fission.

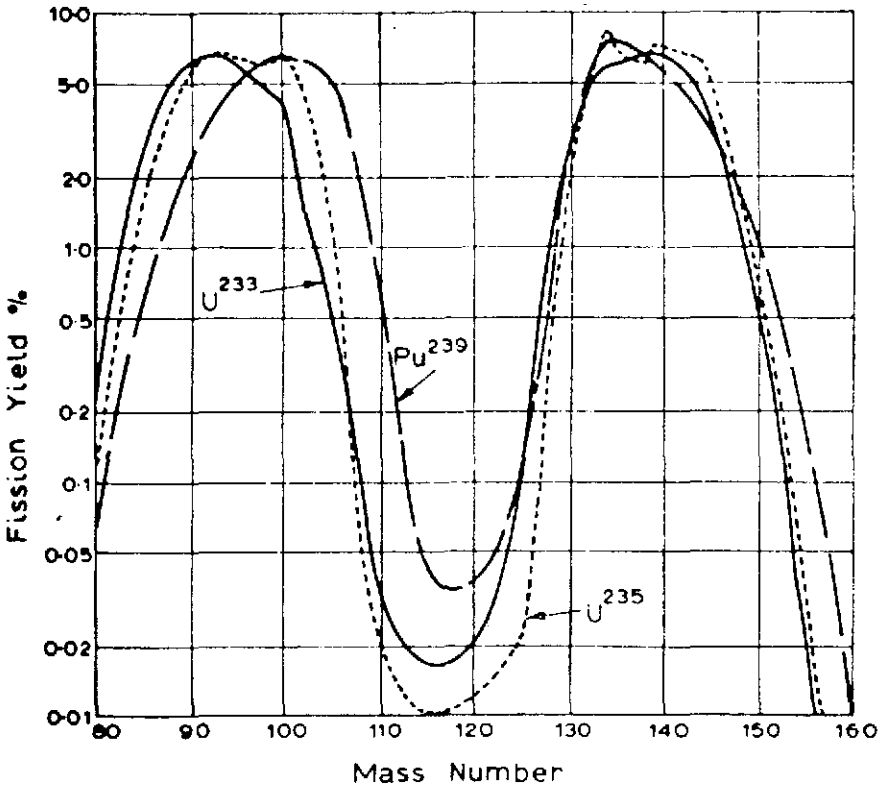


Fig. 1. Fission Yield of U-233, U-235 and Pu-239

Fig. 1 shows the relative frequency with which nuclides of specific mass numbers are produced as fission fragments. Such a curve is known as a *fission yield curve* (since two fragments are produced per fission, the area under the curve adds up to 200%).

You can see that both fission fragments are likely to consist of a substantial piece of the original nucleus. They are likely to have mass numbers between 70 and 160, with those around 95 and 140 being the most probable. Note that symmetrical fission (equal fragments) is quite rare.

Fission Products

The fission fragments are almost invariably radioactive. The reason for this is that the neutron/proton ratio of the fragments is about the same as that of the fissioned nucleus, and this is too high for stability at medium mass numbers. The fragments will therefore try to reduce their n/p ratio by successive β^-,γ decays until stability is reached. A typical decay chain is shown in Fig. 2. All the members of such chains are known as *fission products*.

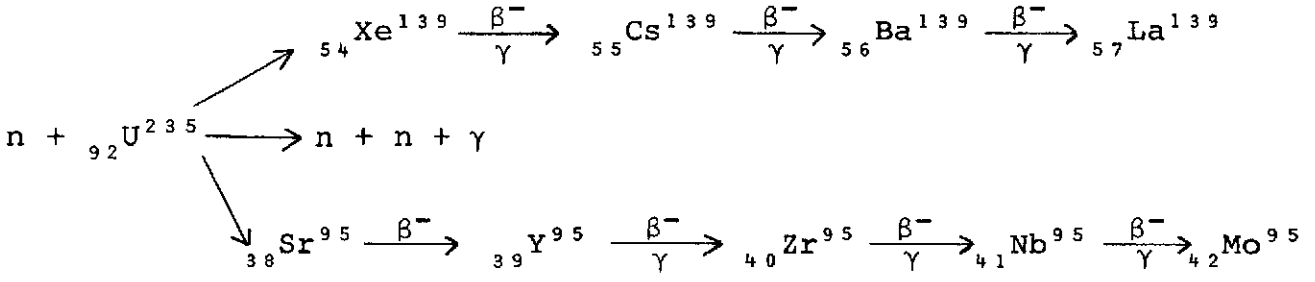


Fig. 2 Fission Product Decay Chain

The half-lives of fission products range from fractions of a second to thousands of years. (It is this activity that causes so much concern in atomic bomb fall-out). There are four important consequences of fission product production in the fuel:-

(a) The fission products must be held in the fuel by encasing it in a sheath, so that they do not enter the heat transport system and hence leave the reactor core. Since many of them have long half-lives, their presence in the heat transport system would be a radiation hazard which would prevent access to equipment even when the reactor is shut down.

(b) Heavy shielding is required around the reactor to avoid exposure to the gamma radiation emitted by the fission products.

(c) Fuel must be changed remotely, and special precautions must be taken in handling and storing spent fuel.

(d) Some of the fission products have a high affinity for neutrons and thereby *poison* the reactor. The two most important poisons are Xe-135 and Sm-149. They are produced in a relatively high percentage of fissions, and they capture a significant number of neutrons.

Prompt and Delayed Neutron Emission

The fission fragments are produced in an excited state and will immediately emit perhaps two or three neutrons and some gamma photons. These are called *prompt neutrons* and *prompt gammas*.

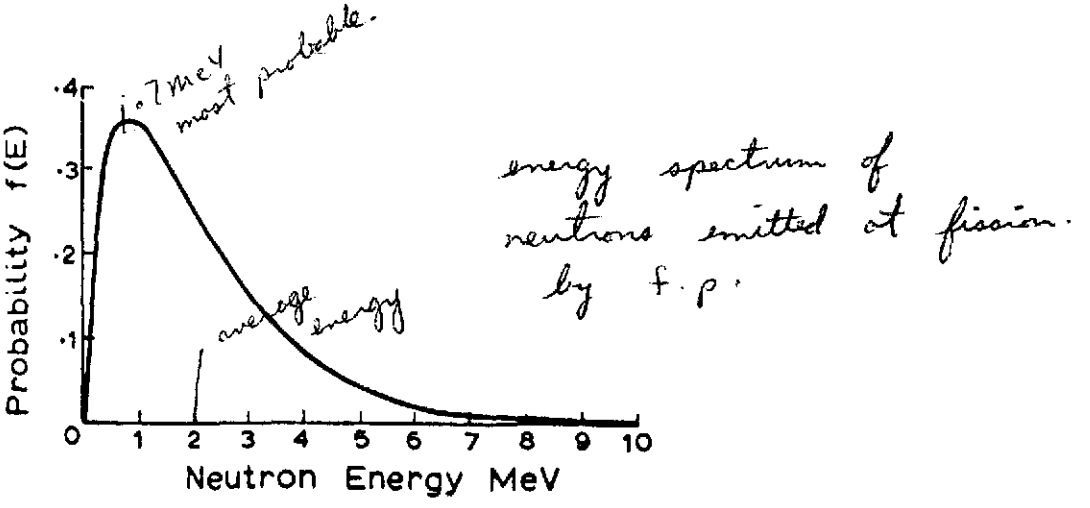


Fig. 3. Prompt Neutron Energy Spectrum

Fig. 3 shows the energy distribution of prompt neutrons. The average energy is about 2 MeV, although the most probable energy is only 0.72 MeV.

A very small number of neutrons (less than 1%) appear long after fission occurs, and these are known as *delayed neutrons*. They arise from the radioactive decay of certain fission product daughters. For example, Bromine-87 decays by β^- emission to Krypton-87. The Krypton-87 formed in this way is sufficiently unstable to be able to emit a neutron to become Kr-86, and in fact it does this more often than not. The neutron emission is instantaneous (with respect to Kr-87), but obviously occurs some time after the original fission because the Br-87 must decay first. In fact, it appears to be emitted with the 55 second half-life of Br-87.

Nuclei such as Br^{87} whose production in fission may eventually lead to the emission of a delayed neutron are known as *delayed-neutron precursors*. At the present time, it is believed that there may be as many as twenty precursors, although only about half a dozen have been positively identified. These precursors and their respective half-lives are given in Table 2. They are usually divided into six groups according to their half-lives.

TABLE 2

Delayed-Neutron Precursors

(Uncertain Quantities are Indicated by Brackets)

Precursor	Half-life and Group	
Br^{87}	54.5	Group 1
I^{137}	24.4	Group 2
Br^{88}	16.3	
I^{138}	6.3	Group 3
$Br^{(89)}$	4.4	
$Rb^{(93,94)}$	6	
I^{139}	2.0	Group 4
(Cs,Sb or Te)	(1.6-2.4)	
$Br^{(90,92)}$	1.6	
$Kr^{(93)}$	~1.5	
(I^{140}+Kr?)	0.5	Group 5
(Br,Rb,As+?)	0.2	Group 6

For thermal fission of U-235, the total contribution of all the delayed neutrons is only 0.65% of the total neutrons produced. With Pu-239, the total delayed fraction is even less at 0.21%. Despite the fact that these fractions are quite small, they have a very important effect on the time dependent behaviour of thermal reactors. We shall discuss this aspect of delayed neutrons in a later lesson on Reactor Control.

Table 3 gives the probability of a particular number of neutrons being emitted in the thermal fission of a U-235 nucleus. This includes both prompt and delayed neutrons.

TABLE 3

Neutron Emission in Thermal Fission of U-235

Number of Neutrons Emitted	Number of Cases per 1000 Fissions
0	27
1	158
2	339
3	302
4	130
5	34

ν = average yield for fission.

The average number of neutrons emitted per fission is an important quantity in reactor physics. It is universally denoted by the Greek letter ν ("new"). For thermal fission of U-235 $\nu = 2.430$. (Fast fissions usually produce marginally more neutrons).

Energy Release From Fission

About 200 MeV of energy is liberated when a nucleus fissions. The exact value slightly depends on the fissile nucleus and on the fission fragments produced. The energy can be calculated as follows:-

Consider the example given in Fig. 2 on page 4:

$$_{92}U^{235} + n \longrightarrow {}_{38}Sr^{95} + {}_{54}Xe^{139} + 2n$$

Total mass before fission	= 235.044	+ 1.009		236.053 amu
Total mass after fission	= 94.903	+ 138.918 + 2.018		= 235.839 amu
			Loss in mass	= 0.214 amu

The energy equivalent of 1 amu is given by:

$$E(\text{joule}) = m(\text{kg}) \times c^2(\text{m.s}^{-1})^2$$
$$= 1.66 \times 10^{-27} \times (3 \times 10^8)^2$$
$$= 1.492 \times 10^{-10} \text{ joule}$$

1 eV = 1.602×10^{-19} joule, so

$$\underline{E = 931.5 \text{ MeV/amu}}$$

For the example considered here, the 0.214 amu mass loss then corresponds to almost exactly 200 MeV. A summary of how this energy might be distributed is given in Table 4.

TABLE 4

Approximate Distribution of Fission Energy Release in U-235

Kinetic energy of lighter fission fragment	100 MeV
Kinetic energy of heavier fission fragment	67 MeV
Energy of prompt neutrons	5 MeV
Energy of prompt γ rays	6 Mev
β particle energy gradually released from fission products	7 MeV
γ ray energy gradually released from fission products	6 MeV
Neutrinos (energy escapes from reactor)	11 MeV
Total	202 MeV

This is not a complete account of all the energy released in the reactor. Some of the neutrons even after losing all their kinetic energy may produce (n,γ) reactions with materials in the reactor, and up to about 8 MeV may be released in such reactions. The total amount of energy produced in a reactor per fission may therefore depend to a slight extent on the form of the reactor, but it is always within a few MeV of 200 MeV.

Reactor Power and Fuel Consumption

The 200 MeV released in one fission is not of much practical value, because it is minute. In fact, 1 watt of power requires 3.1×10^{10} fissions every second.

One Megawatt steady power requires 3.1×10^{16} fissions every second continuously. 3.1×10^{16} atoms of U-235 weigh

$$\frac{3.1 \times 10^{16} \times 235}{6.023 \times 10^{26}} = 1.21 \times 10^{-8} \text{ kg}$$

Therefore, to produce 1 Megawatt-day of energy from fission requires the fissioning of

$$1.21 \times 10^{-8} \times 24 \times 3600 = 1.0 \times 10^{-3} \text{ kg} = \underline{1.0 \text{ g U-235}}$$

The first requirement for producing useful power from the fission process is that enough U-235 nuclei must be available for fissioning. This requirement is met by installing sufficient U-235 in the reactor in the form of fuel rods. If natural uranium is used, of which 0.72% is U-235, then about 140 g of uranium would be used to produce 1 Megawatt-day of energy. This assumes that all the U-235 could be fissioned. In practice this is not so, because some U-235 (~17%) is consumed in (n,γ) reactions. As a result, 165 g of natural uranium would be used.

For example, a Pickering reactor at full power generates 1744 MW from fission (540 MW of electrical power is generated then). It would therefore use about 280 kg of natural uranium a day on this basis. Because Pu-239 (and Pu-241) is produced in the fuel after a while, this contributes substantially to energy production, and the amount of fuel used is consequently smaller, being more like 180 or 200 kg of uranium.
9 fuel bundles.

The second requirement for producing this amount of power continuously is that the rate of fissioning must be maintained. The neutrons released during fission must be used to cause further fissions, so that a *chain reaction* is maintained. How this is achieved is discussed in later lessons.

ASSIGNMENT

1. A fission of U-235 has lead to nuclei of mass numbers 89 and 144. Assuming that the n/p ratio is the same in both fragments, identify the particular nuclides formed and all the fission products that will be produced as a result.

$^{89}_{35}Br$ $^{144}_{57}La$

2. The energy release per fission can also be derived from the binding energy curve in lesson 127.10-1. How?

3. For the same rate of fissioning, why should there be a difference in the energy obtained from fresh fuel and fuel that has been in the reactor for 6 months?

4. The unit used within Ontario Hydro for fuel consumption (called *burn-up*) is MWh/kg. What percentage of the available U-235 is used to produce one MWh from one kg of natural uranium?

5. What is the overall average time delay in the production of delayed neutrons if the yields (neutrons per fission) of groups 1 to 6 are 0.00052, 0.00346, 0.00310, 0.00624, 0.00182 and 0.00066 respectively?

J.U. Burnham

Nuclear Theory - Course 127

NEUTRON CROSS SECTIONS AND NEUTRON FLUX

When a neutron strikes a nucleus, any of the reactions discussed above may take place, depending on the nucleus and the neutron energy. What determines, then, which reaction will occur? In the case of U-238, for instance, inelastic scattering will not occur unless the neutron energy is greater than 0.1 Mev. To put it another way, there is no chance or probability of inelastic scattering occurring with U-238 unless the neutron energy is greater than 0.1 Mev. We could also say that the chance or probability of U-235 fission occurring is greater with thermal neutrons than with fast neutrons, ie, the probability increases as the neutron energy decreases.

Thus we are always comparing the chances in favour of the various reactions taking place. It is the probability of a particular reaction occurring that is important. Some reactions are more probable with some nuclei than with others or more probable with some neutron energies than with others. Because these reactions are concerned with a neutron striking a target, namely a nucleus, the probability that a particular reaction will occur is measured in terms of a quantity called the *nuclear* or *neutron cross section*.

Neutron Cross Sections and Neutron Flux

To examine the precise measuring of the term "cross section", let us look at what happens when n neutrons per unit volume move with velocity v towards a thin target of surface area S. We will assume that the whole target area is exposed to neutrons, and that all the neutrons travel in the same direction x (see Fig. 1).

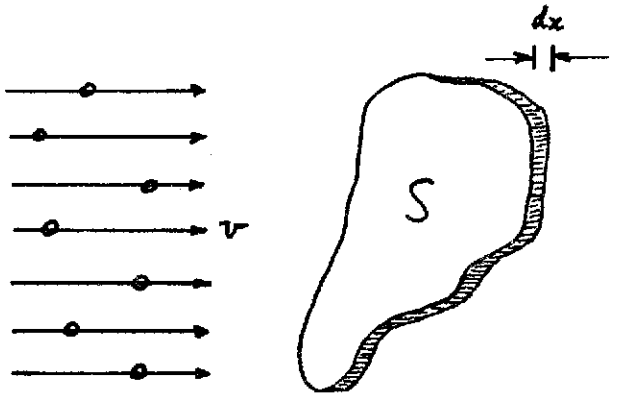


Fig. 1 Neutron Bombardment of a Thin Target

From experiment it is found that the rate R at which a particular reaction occurs is proportional to every one of the following:

(a) $n_x v$, the number of neutrons striking the target in the x direction per unit area and time;

(b) S, the surface area of the target;

(c) dx, the thickness of the target - this is assumed to be sufficiently small for no "shadowing" of the nuclei to occur;

(d) N', a symbol reserved in this course for the number of nuclei per unit volume.

Therefore:

$$R \propto n_x v.N'.Sdx$$

or $$R = \sigma.n_x v.N'.Sdx$$

σ (sigma) is the constant of proportionality, and could be defined as "the interaction rate per atom in the target per unit nv". It is called the *microscopic cross section*, and a little bit of fooling around with units will show that it has dimensions of area. The usual unit is the *barn* (abbreviated b);

$$1\ b = 10^{-28} m^2 = 10^{-24} cm^2 ;$$

it is the same order of magnitude as the physical diameter of a medium size nucleus.

The reaction rate per unit volume of target material is now seen to be:

$$R = n_x v.\ N'\sigma$$

Since N' and σ are both characteristic of the target material, they are often combined to form the:

macroscopic cross section $\Sigma = N'\sigma$

We can now go on to consider neutrons arriving from all directions with the same velocity (see Fig. 2).

For a target of unit volume

$$R\ (total) = N'\sigma\ (n_1 v + n_2 v + \ldots n_i v + \ldots)$$
$$= nv.N'\sigma$$

where n is the *neutron density*, which is the number of neutrons per unit volume regardless of their direction of motion. nv is known as the *neutron flux density* (symbol ϕ and often just called

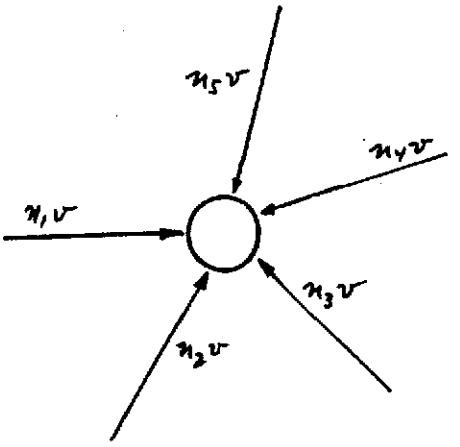


Fig. 2 Isotropic Neutron Bombardment

neutron flux for short). It is usually expressed in units of neutrons.$cm^{-2}s^{-1}$.

The reaction rate for any material exposed to flux ϕ is then:

$R = \phi\Sigma$ per unit volume

Incidentally, it is a common misconception that the neutron flux can be defined as the number of neutrons striking unit area per second. This would be true for a beam, but not for random directions in which case the number hitting unit area would be less (by a factor of 2 actually). If you insist on a connection with area, it can be proved that ϕ is the number of neutrons entering an imaginary sphere each second, of total surface area 4 cm^2 and diametral plane area 1 cm^2.

Another point worth mentioning is that when the neutrons have a range of speeds, an appropriate average cross section is usually chosen. For instance, the detailed structure of the thermal neutron distribution can often be ignored (it certainly will be in this course!), if average thermal cross sections are used.

Since different reactions occur with different probabilities, they will have different cross sections. Throughout this course the following nomenclature will be used:-

σ_f = fission cross section

σ_a = absorption cross section

σ_s = elastic scattering cross section

σ_i = inelastic scattering cross section

In those few cases where $\sigma_f \neq 0$, both fission and radiative capture involve a complete absorption of the neutron, and then σ usually includes both reactions, ie, $\sigma_a = \sigma_f + \sigma_{n,\gamma}$.

For your reference, Table 1 on pages 6 and 7 lists the absorption and elastic scattering cross sections for thermal neutrons only (cross sections usually are strongly energy dependent). We can already arrive at some interesting conclusions by taking a look at these.

(a) Water (H_2O) is a better scatterer of neutrons than heavy water (D_2O) or graphite (carbon), but it is also a much heavier absorber than either. This has important implications in choosing a moderator.

(b) Boron and cadmium have very high values of σ_a and therefore are excellent materials when neutron absorption is required, as, for example, in control rods of a reactor.

(c) The capture cross section of zirconium is much smaller than that of iron. This explains the use of zirconium alloys instead of steel for pressure tubes and fuel sheathing in our reactors.

To appreciate the significance of these cross sections, let us look at a typical problem:

Cobalt-60 gamma sources for radiation therapy units are produced by irradiating cobalt pellets in reactors. A typical pellet might be ¼" in diameter and 1" long. Calculate the activity in curies built up in one of these pellets after it has been irradiated for two years in a thermal neutron flux of $5 \times 10^{13}\ n.cm^{-2}s^{-1}$.

All the data required to solve this problem is already given in the Chart of the Nuclides at the end of the first lesson, and in Table 1 of this lesson (page 6):-

Natural cobalt is 100% Co-59; half-life of Co-60 = 5.3 y; σ_a of Co-59 = 37 b; $\rho = 8.8\ gcm^{-3}$. X use 5.2

We must first write down the differential equation relating Co-60 production and decay per unit volume, ie,

$$\frac{dc}{dt} = \phi\Sigma_a - c\lambda,$$

where c is the concentration of Co-60, λ its ~~half-life~~ decay constant, and Σ_a the macroscopic absorption cross section of Co-59. Solving this equation yields:

$$c = \frac{\phi\Sigma_a}{\lambda}\,(1 - e^{-\lambda t}).$$

In other words, the cobalt activity per unit volume is:

$$c\lambda = \phi\Sigma_a(1 - e^{-\lambda t})$$

$$= \phi N'\sigma_a(1 - e^{-\lambda t})$$

With the substitution of the values $\phi = 5 \times 10^{13} n.cm^{-2}s^{-1}$, $N' = \frac{No}{A}\rho = 9 \times 10^{22}$ atoms cm^{-3}, $\sigma_a = 37 \times 10^{-24} cm^2$, $\lambda = \frac{0.69}{5.2} y^{-1}$ and $t = 2$ y, we get:

$$\underline{c\lambda = 3.9 \times 10^{13} \text{ cm}^{-3}\text{s}^{-1}}$$

The activity has to be in units of $cm^{-3}s^{-1}$, because

$$(cm^{-2}s^{-1}) \times (cm^{-3}) \times (cm^2) = cm^3s^{-1}.$$

To find the activity of the whole pellet in curies, we multiply by the volume and divide by $3.7 \times 10^{10} s^{-1}$ Ci, ie,

$$\text{Activity} = \frac{3.9 \times 10^{13} \times \pi(0.25 \times 2.5)^2 \times 2.5}{4 \times 3.7 \times 10^{10}} \text{ Ci}$$

$$= \underline{810 \text{ Ci}}$$

Actually, the activity will be a bit less than this because of the *self-shielding* of the cobalt pellets. The flux at the centre of the pellet will be less than at the outside, because some neutrons have been removed by absorption. We shall consider this next.

Attenuation of Neutrons

Consider Fig.1 again. After traversing the thickness dx, some neutrons have been removed from the beam. The neutron density will be reduced by an amount dn given by:

$$\frac{dn}{n} = -N'\sigma \, dx;$$

if the target is of thickness x, the neutron density at x is given by:

$$\int_{n_o}^{n_x} \frac{dn}{n} = \int_0^x N'\sigma \, dx$$

or $$\underline{n_x = n_o e^{-N'\sigma x} = n_o e^{-\Sigma x}}$$

TABLE 1

Properties of the Elements and Certain Molecules

Element or molecule	Symbol	Atomic number	Atomic or molecular weight*	Nominal density, gm/cm³	Atoms or molecules per cm³†	σ_a,‡ barns	σ_s,‡ barns	Σ_a,† cm^{-1}	Σ_s,† cm^{-1}
Actinium	Ac	89	227		N'	800			
Aluminum	Al	13	26.9815	2.699	0.06024	0.235	1.4	0.01416	0.08434
Antimony	Sb	51	121.75	6.62	0.03275	5.5	4.3	0.1801	0.1408
Argon	Ar	18	39.948	Gas		0.63	1.5		
Arsenic	As	33	74.9216	5.73	0.04606	4.5	6	0.2073	0.2764
Barium	Ba	56	137.34	3.5	0.01535	1.2	8	0.01842	0.1228
Beryllium	Be	4	9.0122	1.85	0.1236	0.0095	7.0	0.001174	0.8652
Beryllium oxide	BeO		25.0116	2.96	0.07127	0.0095	6.8	0.0006771	0.4846
Bismuth	Bi	83	208.980	9.80	0.02824	0.034	9	0.0009602	0.2542
Boron	B	5	10.811	2.3	0.1281	759	4	97.23	0.5124
Bromine	Br	35	79.909	3.12	0.02351	6.7	6	0.1575	0.1411
Cadmium	Cd	48	112.40	8.65	0.04635	2450	7	113.6	0.3245
Calcium	Ca	20	40.08	1.55	0.02329	0.43	3.0	0.01002	0.06987
Carbon (graphite)**	C	6	12.01115	1.60	0.08023	0.0034	4.8	0.0002728	0.3851
Cerium	Ce	58	140.12	6.78	0.02914	0.7	9	0.02040	0.2623
Cesium	Cs	55	132.905	1.9	0.008610	30	20	0.2583	0.1722
Chlorine	Cl	17	35.453	Gas		33	16		
Chromium	Cr	24	51.996	7.19	0.08328	3.1	3	0.2582	0.2498
Cobalt	Co	27	58.9332	8.8	0.08993	37	7	3.327	0.6295
Columbium (see niobium)									
Copper	Cu	29	63.54	8.96	0.08493	3.8	7.2	0.3227	0.6115
Deuterium	D	1	2.01410	Gas		0.0005			
Dysprosium	Dy	66	162.50	8.56	0.03172	940	100	29.82	3.172
Erbium	Er	68	167.26	9.16	0.03203	160	15	5.125	0.4805
Europium	Eu	63	151.96	5.22	0.02069	4300	8	88.97	0.1655
Fluorine	F	9	18.9984	Gas		0.0098	3.9		
Gadolinium	Gd	64	157.25	7.95	0.03045	46,000	4	1401	0.1218
Gallium	Ga	31	69.72	5.91	0.05105	3.0		0.1532	
Germanium	Ge	32	72.59	5.36	0.04447	2.4	3	0.1067	0.1334
Gold	Au	79	196.967	19.32	0.05907	98.8	9.3	5.836	0.5494
Hafnium	Hf	72	178.49	13.36	0.04508	105	8	4.733	0.3606
Heavy water††	D_2O		20.0276	1.105	0.03323	0.0010	13.6	3.323×10^{-5}	0.4519
Helium	He	2	4.0026	Gas		$\leq$0.050	0.8		
Holmium	Ho	67	164.930	8.76	0.03199	65		2.079	
Hydrogen	H	1	1.008665	Gas		0.332			
Illinium (see promethium									
Indium	In	49	114.82	7.31	0.03834	194	2.2	7.438	0.08435
Iodine	I	53	126.9044	4.93	0.02340	6.4	3.6	0.1498	0.08242
Iridium	Ir	77	192.2	22.5	0.07050	460		32.43	
Iron	Fe	26	55.847	7.87	0.08487	2.53	11	0.2147	0.9336
Krypton	Kr	36	83.80	Gas		24	7.2		
Lanthanum	La	57	138.91	6.19	0.02684	8.9	15	0.2389	0.4026
Lead	Pb	82	203.973	11.34	0.03348	0.17	11	0.005692	0.3683
Lithium	Li	3	6.939	0.53	0.04600	71	1.4	3.266	0.0644
Lutetium	Lu	71	174.91	9.74	0.03354	80		2.683	
Magnesium	Mg	12	24.312	1.74	0.04310	0.063	4	0.002715	0.1724
Manganese	Mn	25	54.9380	7.43	0.08145	13.3	2.3	1.083	0.1873
Mercury	Hg	80	200.59	13.55	0.04068	360	20	14.64	0.8136
Molybdenum	Mo	42	95.94	10.2	0.06403	2.6	7	0.1665	0.4482
Neodymium	Nd	60	144.24	6.98	0.02914	50	16	1.457	0.4662
Neon	Ne	10	20.183	Gas		0.032	2.4		
Nickel	Ni	28	58.71	8.90	0.09130	4.6	17.5	0.4200	1.597
Niobium	Nb	41	92.906	8.57	0.05555	1.1	5	0.06111	0.2778
Nitrogen	N	7	14.0067	Gas		1.85	10		

Element or molecule	Symbol	Atomic number	Atomic or molecular weight*	Nominal density, gm/cm³	Atoms or molecules per cm³†	σ_a,‡ barns	σ_s,‡ barns	Σ_a,† cm^{-1}	Σ_s,† cm^{-1}
Osmium	Os	76	190.2	22.5	0.07124	15	11	1.069	0.7836
Oxygen	O	8	15.9994	Gas		<0.0002	4.2		
Palladium	Pd	46	106.4	12.0	0.06792	8	3.6	0.5434	0.2445
Phosphorus (yellow)	P	15	30.9738	1.82	0.03539	0.19	5	0.006724	0.1770
Platinum	Pt	78	195.09	21.45	0.06622	10	10	0.6622	0.6622
Plutonium	Pu	94	239	19.6	0.04939	σ_a = 1015	9.6	49.88	0.4741
						σ_f = 741		36.55	
Polonium	Po	84	210	9.51	0.02727				
Potassium	K	19	39.102	0.86	0.01325	2.1	1.5	0.02783	0.01988
Praseodymium	Pr	59	140.907	6.78	0.02898	12	4	0.1965	0.1159
Promethium	Pm	61							
Protactinium	Pa	91	231			210			
Radium	Ra	88	226	5.0	0.01332	20		0.2664	
Rhenium	Re	75	186.2	20	0.06596	85	14	5.607	0.9234
Rhodium	Rh	45	102.905	12.41	0.07263	155	5	11.26	0.3632
Rubidium	Rb	37	85.47	1.53	0.01078	0.73	12	0.007869	0.1294
Ruthenium	Ru	44	101.07	12.2	0.07270	2.5	6	0.1818	0.4362
Samarium	Sm	62	150.35	6.93	0.02776	5800	5	161.0	0.1388
Scandium	Sc	21	44.956	2.5	0.03349	23	24	0.7703	0.8038
Selenium	Se	34	78.96	4.81	0.03669	12	11	0.4403	0.4036
Silicon	Si	14	28.086	2.33	0.04996	0.16	1.7	0.1164	0.08493
Silver	Ag	47	107.870	10.49	0.05857	63	6	3.690	0.3514
Sodium	Na	11	22.9898	0.97	0.02541	0.53	4	0.01347	0.1016
Strontium	Sr	38	87.62	2.6	0.01787	1.3	10	0.02323	0.1787
Sulfur (yellow)	S	16	32.064	2.07	0.03888	0.52	1.1	0.2022	0.04277
Tantalum	Ta	73	180.948	16.6	0.05525	21	5	1.160	0.2763
Technetium	Tc	43	99			22			
Tellurium	Te	52	127.60	6.24	0.02945	4.7	5	0.1384	0.1473
Terbium	Tb	65	158.924	8.33	0.03157	46		1.452	
Thallium	Tl	81	204.37	11.85	0.03492	3.3	14	0.1152	0.4889
Thorium	Th	90	232.038	11.71	0.03039	7.4	12.6	0.2249	0.3829
Thulium	Tm	69	168.934	9.35	0.03314	125	7	4.143	0.2320
Tin	Sn	50	118.69	7.298	0.03703	0.63	4	0.02333	0.1481
Titanium	Ti	22	47.90	4.51	0.05670	6.1	4	0.3459	0.2268
Tungsten	W	74	183.85	19.2	0.06289	19	5	1.195	0.3145
Uranium	U	92	238.03	19.1	0.04833	σ_a = 7.6	8.3	0.3673	0.4011
						σ_f = 4.2		0.2030	
Vanadium	V	23	50.942	6.1	0.07212	4.9	5	0.3534	0.3606
Water	H_2O		18.0167	1.0	0.03343	0.664	103	0.02220	3.443
Xenon	Xe	54	131.30	Gas		24	4.3		
Ytterbium	Yb	70	173.04	7.01	0.02440	37	12	0.9208	0.2928
Yttrium	Y	39	88.905	5.51	0.03733	1.3	3	0.04853	0.1120
Zinc	Zn	30	65.37	7.133	0.06572	1.10	3.6	0.07229	0.2366
Zirconium	Zr	40	91.22	6.5	0.04291	0.18	8	0.007724	0.3433

* Based on C^{12} = 12.00000 amu.

† Four-digit accuracy for computational purposes only; last digit(s) usually is not meaningful.

‡ Cross sections at 0.0253 eV or 2200 m/sec. The scattering cross sections, except for those of H_2O and D_2O, are measured values in a thermal neutron spectrum and are assumed to be 0.0253 eV values because σ_s is usually constant at thermal energies. The errors in σ_s tend to be large, and the tabulated values of σ_s should be used with caution. (From BNL-325, 2nd ed., 1958 plus supplements 1 and 2, 1960, 1964, and 1965.)

** The value of σ_a given in the table is for pure graphite. Commercial reactor-grade graphite contains varying amounts of contaminants and σ_a is somewhat larger, say, about 0.0048 barns, so that $\Sigma_a \approx 0.0003851$ cm^{-1}.

†† The value of σ_a given in the table is for pure D_2O. Commercially available heavy water contains small amounts of ordinary water and σ_a in this case is somewhat larger.

Table and data reprinted from Lamarsh: "Introduction to Nuclear Reactor Theory" by permission of Addison-Wesley Publishing Co. Inc.

This shows that penetration through a distance $x = 1/\Sigma$ reduces the neutron density by a factor of e. It can be shown that this distance $1/\Sigma$ is the average distance a neutron will travel before interacting. This result does not only apply to a beam, but is quite general. The distance $1/\Sigma$ is called the *mean free path*, and is given the symbol λ. Before applying this to a problem on mean free paths in fuel, let us list the thermal neutron cross sections of fuel atoms in Table 2 (the values of ν are given for the sake of completeness). We shall make extensive use of this data later in the course.

TABLE 2

Thermal Neutron Cross Sections of Fuel Atoms (in Barns)

(taken from Atomic Energy Review (IAEA), 1969, Vol 7, No 4, p 3)

	σ_f	$\sigma_{n,\gamma}$	σ_a	σ_s	ν
U-233	530.6	47.0	577.6	10.7	2.487
U-235	580.2	98.3	678.5	17.6	2.430
U-238	0	2.71	2.71	~10	0
nat.U	4.18	3.40	7.58	~10	
Pu-239	741.6	271.3	1012.9	8.5	2.890
Pu-241	1007.3	368.1	1375.4	12.0	2.934

Example: Calculate the absorption mean free path of thermal neutrons in natural uranium.

$$\lambda_a = \frac{1}{\Sigma_a} = \frac{1}{\Sigma_f + \Sigma_{n,\gamma}} = \frac{1}{N'(\sigma_f + \sigma_{n,\gamma})}$$

Using the data given in tables 1 and 2, we see that:

$$\lambda_a = \frac{1}{0.048 \times 10^{24} \times 7.58 \times 10^{-24}} \text{ cm}$$

$$= \underline{2.08 \text{ cm}}$$

Incidentally, this rather small value of λ_a helps to explain why the neutron flux at the centre of a fuel bundle is significantly smaller than at its perimeter, giving rise to a so-called *flux depression*.

ASSIGNMENT

1. Prove that the mean free path $\lambda = 1/\Sigma$ for any reaction.

2. U-238 has a very high absorption (σ_a = 8000b) for neutrons of 6.5 eV energy. What is the probability of such neutrons surviving capture in traversing natural uranium of 0.1 mm thickness?

3. Calculate the number of fission neutrons emitted per thermal neutron absorbed in natural uranium and uranium enriched in U-235 to 2% and 10%.

4. A useful expression relating the total thermal power P generated in a reactor to the average neutron flux $\bar{\phi}$ and the quantity of natural UO_2 fuel M is given by:

$$P = \frac{\bar{\phi}.M}{3 \times 10^{12}}$$

where P is in MW, ϕ in $n.cm^{-2}s^{-1}$ and M in Mg. The density of UO_2 is 10.7 $g.cm^{-3}$. Derive this expression.

5. The neutron detectors used in Pickering start up were He-3 proportional counters. They are about 12" long and 2" in diameter, and are filled with He-3 gas at 10 atomospheres. Calculate the expected count rate per unit neutron flux assuming that each neutron reacting in the counter volume will be registered. Also explain why the actual count rate should be less than this, even if the above assumption were valid.

He-3(n,p)H-3 reaction cross-section = 5400 b,
$N_o = 0.6 \times 10^{24}$ atoms per 22400 cm^3 at standard temperature and pressure.

6. ~~The predominant~~ Part of the activity in the primary coolant during reactor operation is due to O^{19}. Show that the specific activity ($dis.s^{-1}cm^{-3}$) of N^{16} in the coolant as it leaves the core is given by:

$$A = \frac{\Sigma\phi(1-e^{-\lambda t})}{1-e^{-\lambda T}}$$

$O^{18} + n \rightarrow O^{19} + \gamma$

where t is the core transit time and T the total circuit time.

$\sigma_a\ O^{18} = .21$ mb.

Calculate this activity for the Douglas Point reactor, for which $\phi = 3 \times 10^{13}$ n.cm^{-2}s^{-1}, t = 0.8s, T = 12.7s and D_2O density = 0.842 g cm^{-3} at operating temperature.

J.U. Burnham

Nuclear Theory - Course 127

THE CHAIN REACTION

We have already seen that there are two requirements for the useful production of power from a reactor:

1. the rate of fissioning must be high, and
2. it must be maintained continuously.

The first requirement can be met by having sufficient U-235 nuclei available in the reactor; this implies a sufficiently large quantity of fuel. The second requirement can be met if the neutrons produced at fission are made to cause further fissions, and in this way maintain a chain reaction. In this lesson, we shall consider some of the conditions which are necessary for such a chain reaction to be maintained.

Chain Reactions in Natural Uranium

To repeat the fission process over and over again, a continuous supply of neutrons must be available and the only source of new neutrons is the fission process itself. The fission process could be repeated indefinitely if some, or all, of these neutrons, produced at fission, could be used to produce further fissions. The fission process would then become self-sustaining and a chain reaction results. The minimum condition, under which a chain reaction can be maintained, is that one neutron, produced at fission, be available to cause further fission. Since 2½ new neutrons are produced during each fission, it would seem relatively easy to be able to use one of these neutrons to cause a further fission.

However, in practice it is found that if one neutron from each fission is to be available to cause another fission, a careful choice of reactor material and design is required. This is perhaps most easily illustrated by considering the feasibility of maintaining a chain reaction in natural uranium.

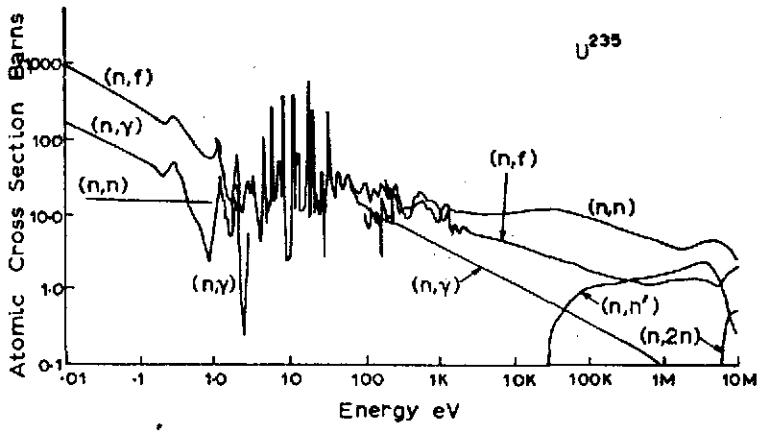


Fig.1 Macroscopic Cross Sections of U-235

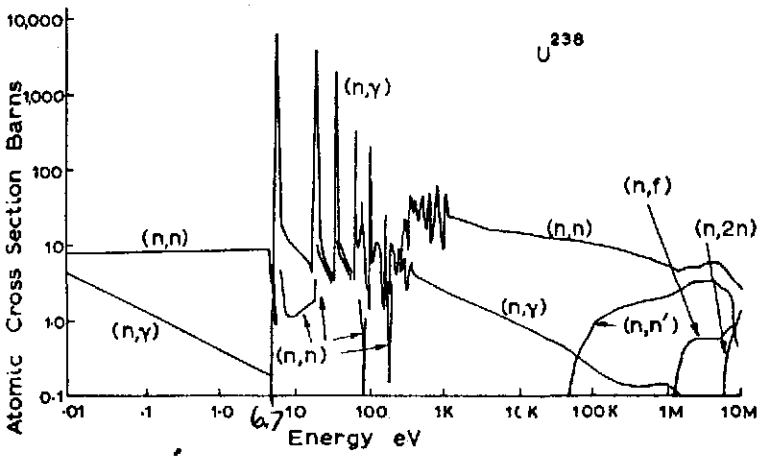


Fig.2 Macroscopic Cross Sections of U-238

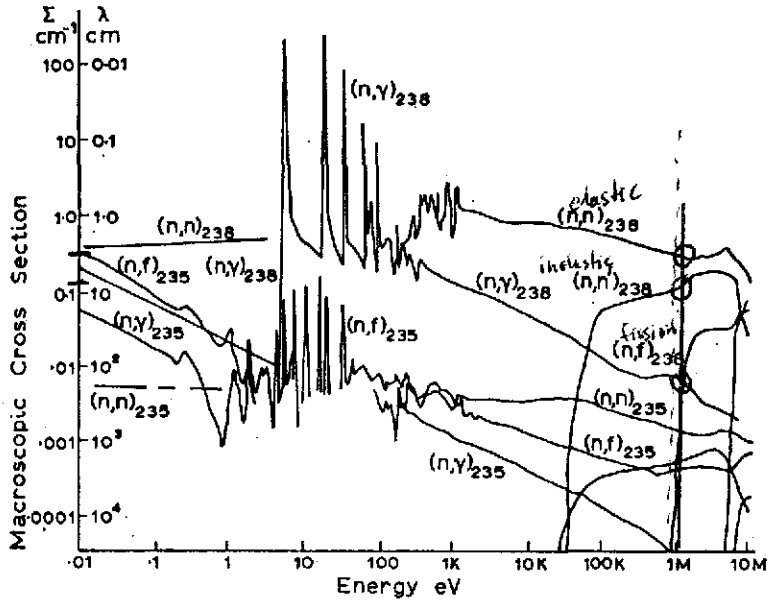


Fig.3 Macroscopic Cross Sections of Natural Uranium

n, n′ inelastic scatter
n, γ rad. capt
n, n elastic scatter

Figures 1 and 2 show the macroscopic cross sections of the various possible interactions with the two isotopes. In considering what happens in a block of natural uranium, these cross sections have to be weighted according to the relative abundances of the two isotopes, and this can be done by considering the macroscopic cross sections as shown in fig.3.

Below the U-238 fission threshold (~1 MeV) a neutron can only produce fission in U-235, but this is relatively unlikely as can be seen from fig.3. Above 1 MeV, fission in U-238 is possible, although not as likely as a scattering collision.

From the cross sections shown in fig.3 it is possible to obtain an indication of the fate of, say, 100 neutrons which are born in the fission process inside a large block of natural uranium. From the prompt neutrons energy spectrum (fig.4, repeated here from lesson 127.10-3), you can see that about 70 of the

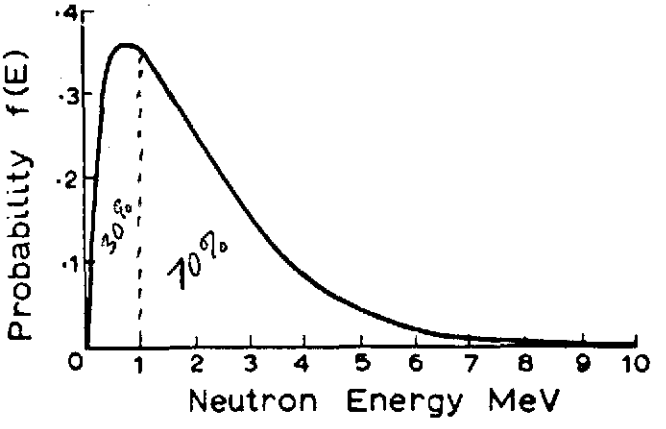


Fig.4 Prompt Neutron Energy Spectrum

neutrons are born with energies above the U-238 fission threshold, and about 30 have energies below the threshold. We shall consider these 30 neutrons first.

From fig.3 you can see that most of the neutrons are elastically scattered, ie, $(n,n)_{238}$. Such collisions change the direction of the neutron and reduce its energy, although the reduction is small because of the large difference in mass between the uranium nucleus and the neutron. But you already know all this anyway. Inelastic scattering collisions, ie, $(n,n')_{238}$, are still possible at neutron energies down to about 100 keV, and in these collisions the neutron loses a considerable

amount of energy since the uranium nucleus is left in an excited state. Therefore, after many collisions the neutrons will all be in an energy region below 100 keV and thereafter lose energy very slowly in scattering collisions. Eventually they will make a collision with a uranium atom to be absorbed, ie, $(n,\gamma)_{238}$. Since in this region the probability of capture by U-238 exceeds the probability of fission with U-235 by a factor of about 40, rather less than 1 neutron out of the original 30 produces fission - the rest are all captured by U-238.

Consider now the other 70 neutrons which are born above the fission threshold. They will undergo the various possible reactions in direct proportion to the corresponding cross sections. Therefore, from fig. 3 you can see that in their first collision about 38 neutrons are elastically scattered, in which case the neutron energy still remains above the threshold. About 27 neutrons are inelastically scattered, and for the purpose of this illustration we can assume that the loss in neutron energy is sufficient to remove them to an energy range below the fission threshold. About 4 neutrons out of the 70 undergo fission in U-238. and the remaining neutron to be accounted for is assumed to be captured in U-238, although there is also a small probability that it interacts with U-235.

The 38 neutrons which are elastically scattered make a second collision in the uranium, and the various reactions are again possible. About 2 neutrons produce fission, about 15 are inelastically scattered to an energy region below the threshold, and about 21 neutrons are elastically scattered to remain above the threshold and make further collisions.

If this sort of argument is continued to the bitter end, you will see that out of the 100 fission neutrons about 8 produce fission in U-238, and about 2 produce fission in U-235 at energies below about 0.1 MeV. Out of the original 100 neutrons, all but 10 suffer non-fission capture in U-238. If we assume that $\nu = 2.5$, 25 second generation neutrons will be produced. The *multiplication factor**) is therefore $k = 0.25$. This is well below the value of 1.0 which is necessary to maintain a chain reaction.

---

*) The multiplication factor, k, can be defined by

$$k = \frac{\text{the number of neutrons in one generation}}{\text{the number of neutrons in the previous generation}}$$

Systems Which Will Produce Chain Reactions

Although the above reasoning undoubtedly indicates that it is not possible to produce a chain reaction in natural uranium, it does suggest ways of obtaining a chain reaction. One way is to increase the relative amount of U-235 to U-238, that is to *enrich* the fuel in the U-235 so that the chance of fission in U-235 at energies below 1 MeV is now increased. If the enrichment is such that out of the 88 neutrons which enter this energy region at least 32 produce fission in U-235, a chain reaction can be maintained. In this case, out of the original 100 neutrons 8 produce fission in U-238, 32 in U-235 and most of the remainder are captured by U-238. These 40 fissions give rise to 40 x 2.5 100 second generation neutrons and the multiplication factor is then unity.

Fast Reactions

A reactor in which most of the fissions take place at energies above 100 keV is called a *fast reactor*. Fast reactors are described in some detail in the Reactor Boiler & Auxiliaries Course 133, and so only the barest outlines will be given here. Fast reactors require relatively highly enriched uranium or plutonium fuel (at least 20%) to increase the U-235 fast fissions. (It can be shown that about 7% enrichment of U-235 in uranium metal is the lowest possible concentration for criticality.) Furthermore, since the fission cross sections are low at high energies, a large concentration of the fuel is required to give a critical assembly - as a general rule, about half the core volume would be fuel, the rest being taken up by heat transfer and structural materials. Since enriched fuels are expensive, a high fuel concentration requires a high power density in order to bring the cost of the power down to competitive levels. Power densities in the region of a kilowatt/cm^3 are necessary, and in order to remove the heat a very efficient means of heat transfer is required, usually a liquid metal.

The production of enriched fuel is difficult and is an expensive process since it cannot be done by chemical means. Plants have been built in the U.S.A., England, France, U.S.S.R. and China using the gaseous diffusion process in which UF_6 diffuses through a porous barrier. The lighter isotope diffuses slightly faster than the heavier, and after many stages the two isotopes can be separated. The three enrichment plants in the U.S.A. between them require almost 5000 MW of electrical power.

Thermal Reactors

Because of the difficulty and expense involved in the production of enriched fuel, reactors using natural uranium have been built by adding material of low atomic mass to the uranium. The purpose of this is to cause the neutrons to lose energy by elastic scattering collisions. Provided that the material of light mass does not have appreciable absorption, the neutrons will be slowed to an energy in the region of 0.025 eV where they are in thermal equilibrium with the atoms of the material. At this energy the probability of producing fission in U-235 is high (580 b compared with 1.2 b at 1 MeV), whereas that of (n,γ) capture in U-238 is not (2.7 b). Even allowing for the low U-235 content of natural uranium, fission is now more likely than capture.

Materials of low atomic mass which do not absorb neutrons to a significant extent are called *moderators*. Reactors which contain large masses of moderating material and in which most of the neutrons reach thermal equilibrium with the moderating atoms are called *thermal reactors*.

ASSIGNMENT

1. It was said in this lesson that uranium with a minimum concentration of about 7% U-235 is necessary before a chain reaction can be maintained. Support this statement with an argument similar to that used in the text.

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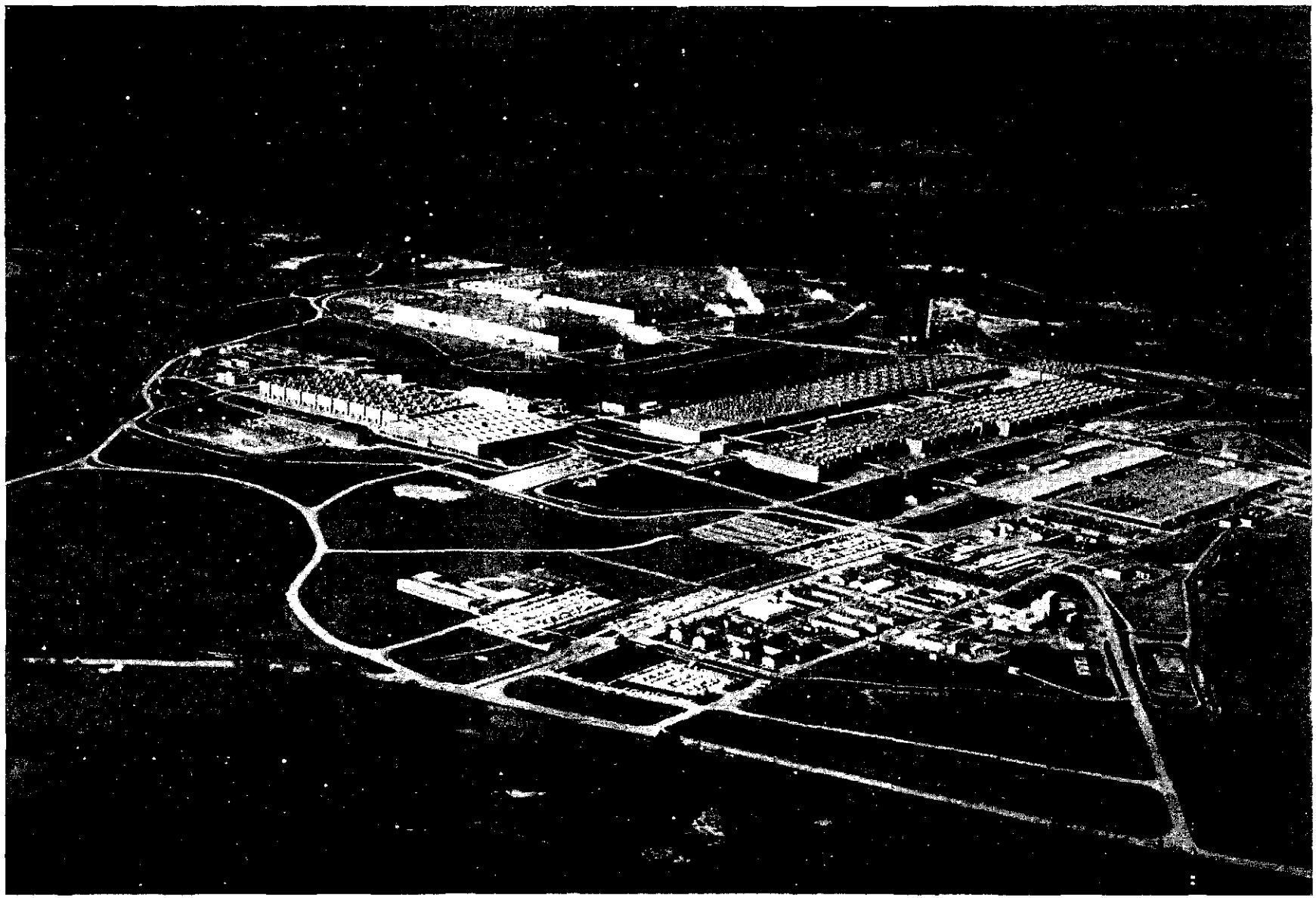


Fig. 5 Gaseous Diffusion Plant at Oak Ridge, Tennessee
(Courtesy of USAEC)

Nuclear Theory - Course 127

NEUTRON BALANCE AND THE FOUR FACTOR FORMULA

When a reactor is operating at steady power, the chain reaction is just being maintained. One neutron only is available from each fission to cause a further fission. The reactor is then said to be just critical, with the neutron population remaining constant.

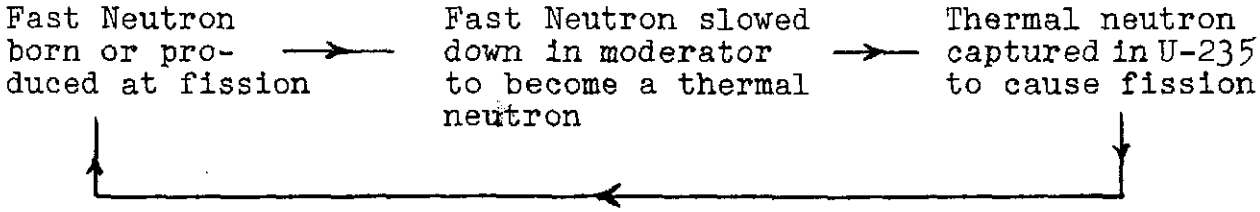
If neutron losses, by leakage or radiation capture, are reduced further, then more neutrons are available to cause fission. The number of fissions occurring in any one generation will be greater than in the previous generation. There is, therefore, a multiplication of neutrons. The multiplication factor, k, is defined by:-

$$k = \frac{\text{Number of neutrons causing fission in any one generation}}{\text{Number of neutrons causing fission in the previous generation}}$$

Consideration will now be given to the quantities or factors on which k depends. In this lesson the factors themselves will be introduced by discussing the neutron balance in a reactor operating at steady power.

The Neutron Cycle

The neutron from each fission, that causes a further fission to maintain the chain reaction, goes through a typical cycle which is shown in Fig. 1.



Of the 2½ neutrons produced at fission, the only one that goes through this cycle is the one that is used to maintain the chain reaction. The others are lost by capture or escape during this cycle. It is useful to know when and how these neutrons are lost since these losses affect the value of k. A complete neutron cycle will, therefore, be considered. This cycle shown in Fig. 2, is very similar to the one given in the Level 2 course except that symbols are used instead of numbers.

We start the cycle with N thermal neutrons in the reactor, and go round the cycle step by step as follows. The continuous lines indicate steps that contribute to the chain reaction and the dotted lines indicate losses of neutrons.

(a) A fraction f, of the thermal neutrons, are assumed to be absorbed in the fuel and a fraction (1 - f) is, therefore, lost by radiative capture in material other than fuel. So fN thermal neutrons are absorbed in the fuel.

The quantity f is called the THERMAL UTILIZATION FACTOR. It could also be defined as the ratio of the neutrons absorbed by the fuel to the total neutrons absorbed in the reactor.

(b) Not all the neutrons absorbed by the fuel will cause fission. Some will be lost by radiative capture. Let a fraction "a" cause fission in U-235, the remainder, (1 - a), being lost by radiative capture. Therefore, afN fissions will occur.

(c) Each fission will produce ν (Greek letter nu) fast neutrons. ν is about 2.5 for U-235. Therefore, a νfN fast neutrons are produced by the original N thermal neutrons.

The product aν is usually replaced by one letter η (Greek letter eta) so that ηfN fast neutrons are produced by U-235 fissions. The factor η now represents the number of fast neutrons produced from each neutron <u>captured</u> in the fuel, whereas ν represents the number of neutrons produced from each neutron actually causing fission.

(d) In (c) we have only considered fast neutrons produced from U-235 fissions, caused by thermal neutron capture. Some of these fast neutrons produced by U-235 fissions, cause U-238 fissions before they are slowed down. Since each U-238 fission produces 2½ new neutrons, the total fast neutron production will be greater than ηfN. This additional contribution is shown by increasing the fast neutron production by a factor ϵ (Greek letter epsilon) ie, the total fast neutrons produced by both U-235 and U-238 fission is $\epsilon\eta$fN.

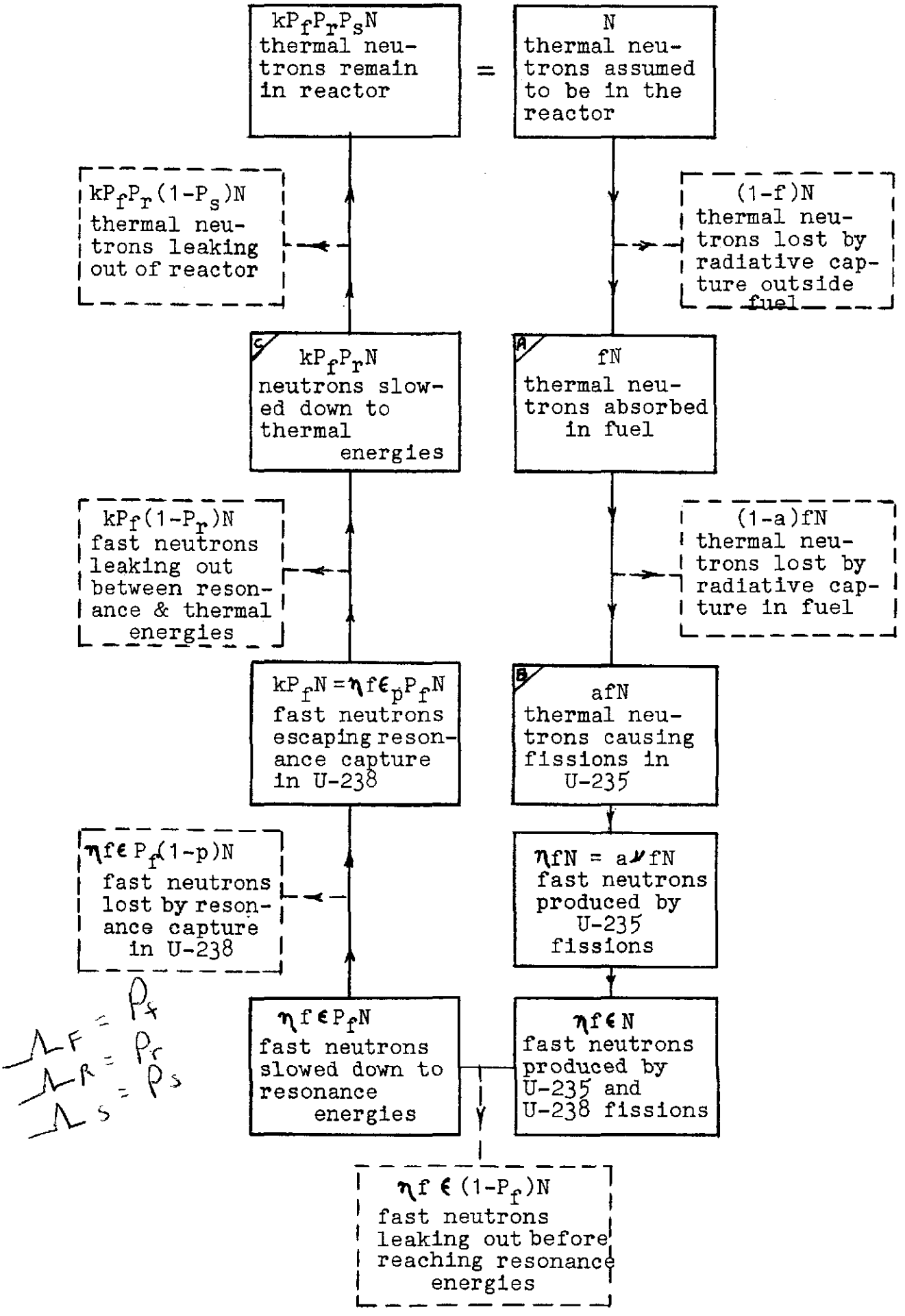
ϵ is known as the FAST FISSION FACTOR.

(e) The fast neutrons produced now have to be thermalized. Some of them escape from the reactor before the resonance energies are reached. If $\epsilon\eta f(1-P_f)N$ escape, the remainder $\epsilon\eta fP_fN$ remain and are slowed down to resonance energies.

P_f, is therefore the fraction of the fast neutrons produced which reach resonance energies without escaping.

(f) If a fraction p now avoids resonance capture, $\epsilon\eta fpP_fN$ neutron are slowed down below resonance energies and the remainder are lost by resonance capture.

Fig. 2 THE NEUTRON CYCLE



$\Lambda_F = P_f$
$\Lambda_R = P_r$
$\Lambda_S = P_s$

(f) This fraction, p, is known as the RESONANCE ESCAPE PROBABILITY.

Neglecting the escaping neutrons we have now allowed $\epsilon\eta fpN$ neutrons to slow down to thermal energies and there were N thermal neutrons originally. So the neutrons have multiplied $\epsilon\eta fp$ times, ie, the neutron multiplication factor, ignoring leakage, is $\epsilon\eta fp$. Thus $k = \epsilon\eta fp$ and kP_fN neutrons slow down below resonance energies.

(g) More neutrons will however escape before thermal energies are reached and only some fraction P_r will become thermalized. Therefore, kP_fP_rN thermal neutrons are produced.

(h) Some thermal neutrons also escape and $kP_fP_rP_sN$ neutron remain in the reactor, P_s being the fraction of the thermal neutrons that do not escape.

The product P_fP_r represents the total fraction of the fast neutrons, which are produced at fission and which do not escape, ie, P_fP_r is the total fast neutron non-leakage probability.

It is not necessary to start the cycle at the particular box chosen. It could be started from the boxes marked A, B or C, for instance. However, if the cycle is to be continuous and the chain reaction just maintained, then on completing the cycle back to the starting box, the same number of neutrons must appear in the box as was there when the cycle was started.

For example a simplified cycle, using numerical values and starting at box A, would be as shown in Fig. 3.

Of 100 thermal neutrons absorbed in U-235 nuclei, 17 suffer radiative capture and the remaining 83 cause fission. From 83 U-235 fissions, 210 fast neutrons are produced, (ie, $\nu = 2.54$). The U-238 fissions are then ignored, since $\epsilon \approx 1$. Nine fast neutrons then leak out of the reactor so that:

$$P_fP_r = \frac{201}{210} \approx 0.96$$

16 fast neutrons suffer resonance capture and $p = \frac{185}{201} = 0.92$.

Thus, 185 thermal neutrons are produced by slowing down, in box C. Of these 10 escape, (ie, $P_s = 175/185 \approx 0.95$), leaving 175. A further 19 thermal neutrons suffer radiative capture outside the fuel, so that:

$$f = \frac{(185 - 10 - 19)}{(185 - 10)} = \frac{156}{175} = 0.89$$

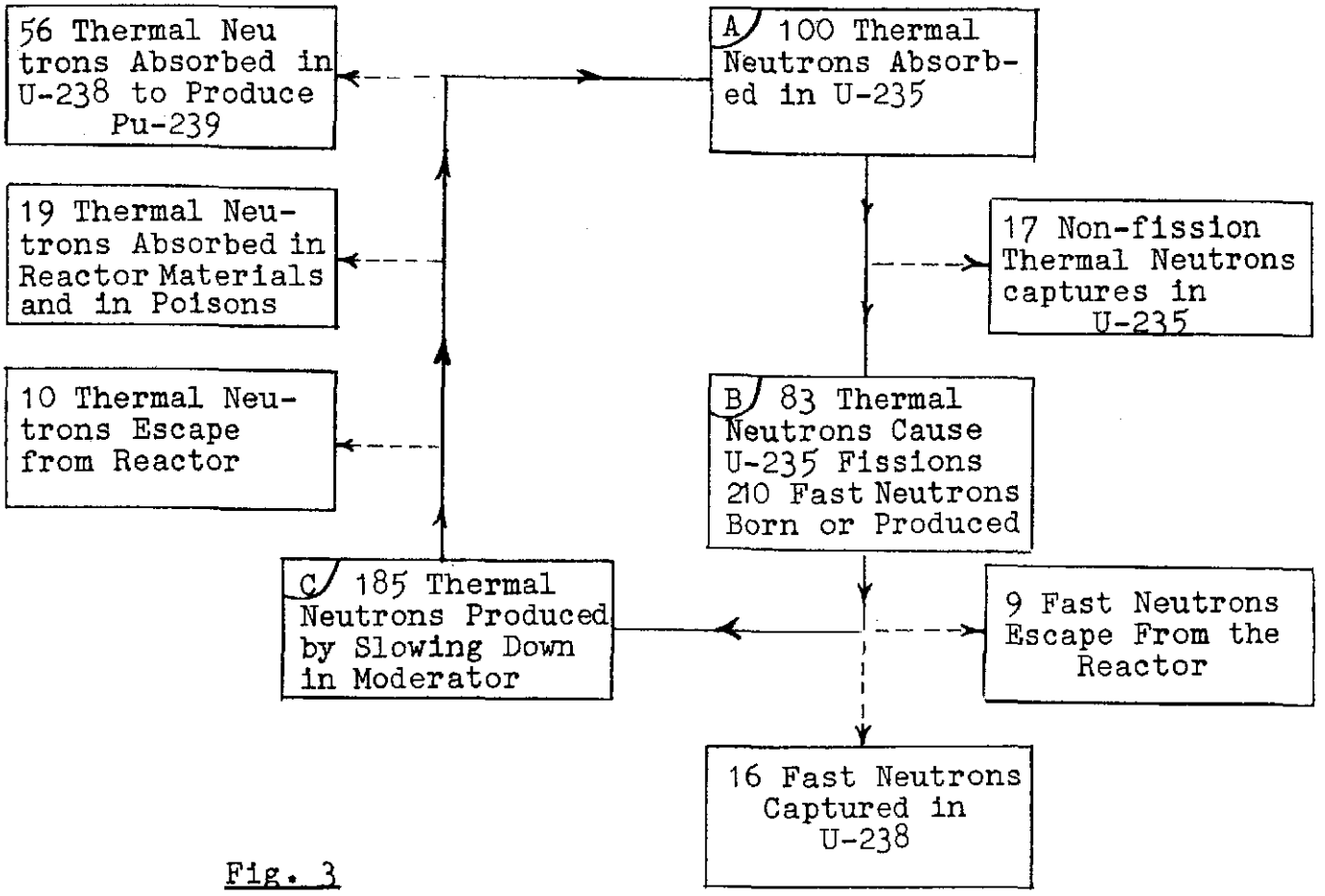


Fig. 3

Finally 56 neutrons suffer radiative capture in U-238, so that the total neutron radiative capture losses in the fuel is 56 + 17.

$$\text{Hence } a = \frac{156 - 73}{156} = \frac{83}{156} = 0.53$$

The value of "a" should, of course, be equal to σ_f / σ_a for natural uranium. Thus, since σ_a = 7.6 barns and σ_f = 4 barns, this ratio should be 0.53.

$$\eta = \frac{210}{156} = 1.3$$

$$\text{also } \eta = a\nu = 2.54 \times 0.53 = 1.35$$

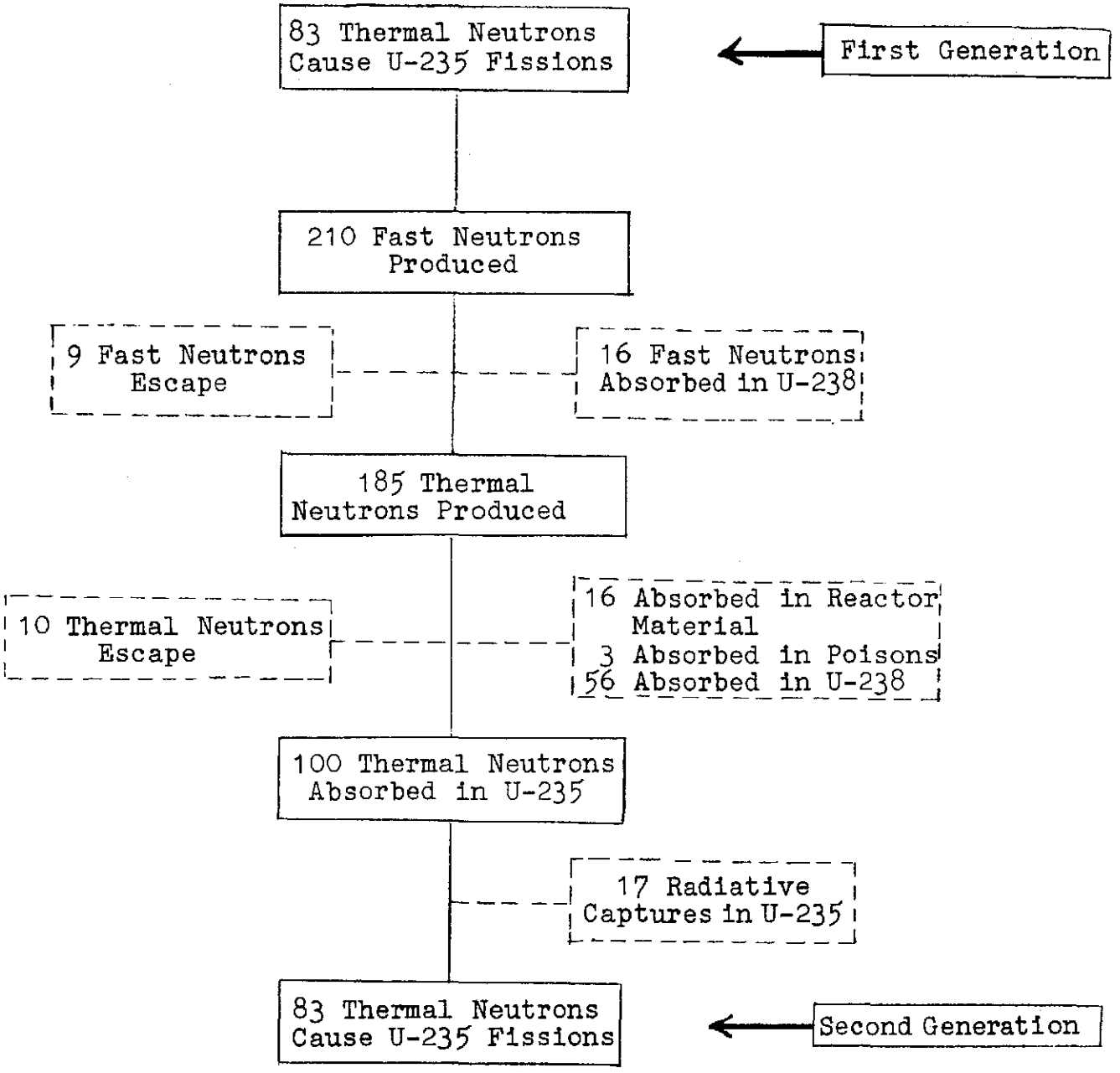


Fig. 4

Alternatively, the cycle could have been started from box B. Fig. 4 shows such a cycle, starting from box B. The cycle has been written down in a somewhat different manner to illustrate the significance of successive generations of neutrons.

Again, fast fissions in U-238 have been ignored to simplify the cycle. The 19 thermal neutrons, lost by radiative capture outside the fuel material have now been subdivided into 16 captured in reactor materials and 3 captured by poisons. The poisons, such as Xe-135, are, of course, produced in the fuel elements but they do not form a part of the fissile or fertile material which are classified as fuel material.

At the end of a further cycle there would be a third generation of neutrons and so on. The number of neutrons generated in each generation is not important. If at the end of each cycle they remain constant, the chain reaction is just being maintained. If the number of fissions in succeeding generations decreases, as would be the case if the neutron losses increased, then the chain reaction cannot be maintained and the reactor power decreases. If the number of neutrons in succeeding generations increases, neutron multiplication occurs and the reactor power increases.

The Four Factor Formula

Returning to the cycle in Fig. 2, the number of neutrons after each cyle will remain constant and the chain reaction will just be maintained if: -

$$kP_fP_rP_sN = N$$

$$\text{or} \quad kP_fP_rP_s = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

This is, then, the condition for criticality

It is much easier, when considering the factors that affect k, to initially ignore all neutron leakage out of the reactor and then allow for it later. Theoretically, the leakage is zero only for a reactor of infinite size but, in practice, it is near enough zero for large reactors. To indicate that an infinitely large system is being considering and that neutron leakage is being ignored, the multiplication factor is written as k_∞ . The condition for criticality, with the chain reaction just being maintained now becomes:

$$k_\infty = \eta \epsilon pf = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

This equation is frequently referred to as the FOUR FACTOR FORMULA. It connects the neutron multiplication factor, k_∞ , with the four factors, η , ϵ , p and f, which determine its value.

If a system of finite size is being considered, the effective multiplication factor, k_e is given by:

$$k_e = k_\infty - \text{neutron leakage}$$

$$\text{or } k_e = \eta \epsilon p f - \text{neutron leakage} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The condition for criticality is, then:

$$k_e = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equations (1) and (4) specify the same condition.

Non-leakage Probabilities

P_f is the fraction of fast neutrons that are reduced to resonance energies without escaping out of the reactor. P_r is the fraction of neutrons which are slowed down from resonance to thermal energies without escaping. The product $P_f P_r$, therefore, represents the total fraction of fast neutrons that do not escape, ie, $P_f P_r$ is the fast neutron non-leakage probability.

$$\text{Now } P_f P_r = e^{-B^2 L_s^2}$$

where B is a quantity called the BUCKLING, which is associated with the flux distribution and L_s is the slowing down length of fast neutrons. This equation shows how L_s determines the fast neutron leakage out of a reactor. The longer the slowing down length the smaller $P_f P_r$ and the greater the leakage. This is another reason why the thermalization should require as few collisions as possible.

P_s is the fraction of thermal neutrons that stay in the reactor, ie, the thermal neutron non-leakage probability.

$$\text{Now } P_s = \frac{1}{1 + B^2 L^2}$$

where L is the diffusion length. If L increases, P_s decreases and thermal neutron leakage increases. So the diffusion length of thermal neutrons should be small.

Equation (1) now becomes: -

$$\frac{k_\infty \, e^{-B^2 L_s^2}}{1 + B^2 L^2} = 1 \quad \text{.....(5)}$$

as the condition for criticality.

Equation (3) can also be more accurately written as

$$k_e = \frac{k_\infty \, e^{-B^2 L_s^2}}{1 + B^2 L^2} = \frac{\eta \epsilon p f \, e^{-B^2 L_s^2}}{1 + B^2 L^2} \quad \text{.....(6)}$$

ASSIGNMENT

1. Define the neutron multiplication factor, k.

2. (a) In what ways can the fast neutron, produced at fission, be lost while being thermalized?

   (b) By what methods, other than leakage, are thermalized neutrons lost so that they are not available for fission?

3. Define or explain the terms "Thermal Utilization Factor", "Fast Fission Factor" and "Resonance Escape Probability".

4. (a) What expression expresses the condition for criticality if leakage is considered?

   (b) Write down the "Four Factor Formula" and explain how it is obtained from the expression in 4a.

5. Explain why the slowing down length and diffusion length of a neutron affect neutron leakage.

A. Williams

Nuclear Theory - Course 127

MODERATOR PROPERTIES

In the previous lesson you learnt that it is possible to sustain a chain reaction either by enriching the fuel in U-235, or by adding a moderator to natural uranium fuel.

Up till now, the second alternative has been by far the most popular one, and the major differences in the design of power reactors have, in the first instance, been dependent on the type of moderator chosen. In this lesson we shall examine the basis on which this choice is made. In other words, this boils down to looking at the requirements of a moderator and then seeing to what extent these are met by the various possible moderator materials.

Neutron Absorption

The primary objective of a moderator is a lot easier to express than to achieve: the fission neutrons must be slowed down to thermal energies without being absorbed. Let us examine the latter aspect first:

There are two possibilities:- the neutrons can be absorbed by the moderator atoms themselves or by fuel atoms, and this can occur anywhere in the energy range from ~2MeV (fission neutrons) down to 0.025 eV (thermal neutrons). Absorption by moderator atoms can obviously be minimized by choosing a moderator with a sufficiently low absorption cross section, but for fuel the argument is rather more subtle.

If you refer back to Fig. 2 of the previous lesson, you will see that U-238 exhibits a number of severe (n,γ) absorption peaks between 5 and 100 eV. These are called *resonances*, and any neutrons absorbed by them are said to have suffered *resonance capture*. It is essential to minimize such resonance capture, and one way of doing this is to ensure that, in the slowing down process, the neutron energy loss per collision is as high as possible. For example, consider the two figures on the following page (for the sake of simplicity the resonances have been smoothed out).

Moderator 2 thermalizes the neutrons in far fewer collisions than moderator 1. This means that the neutrons in moderator 2 will spend less time in the resonance energy region, and will therefore also have less chance of colliding with U-238 while they have this energy. If they do, they will almost certainly be captured, and if you don't believe this have another look at Assignment #2 in the earlier lesson in cross sections. The conclusion of all this then is that there will be less resonance capture in U-238 with moderator 2 than with moderator 1.

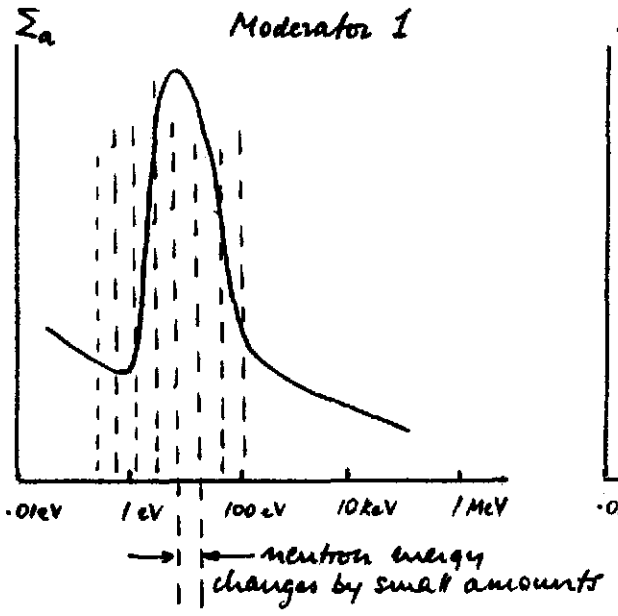


Fig. 1.

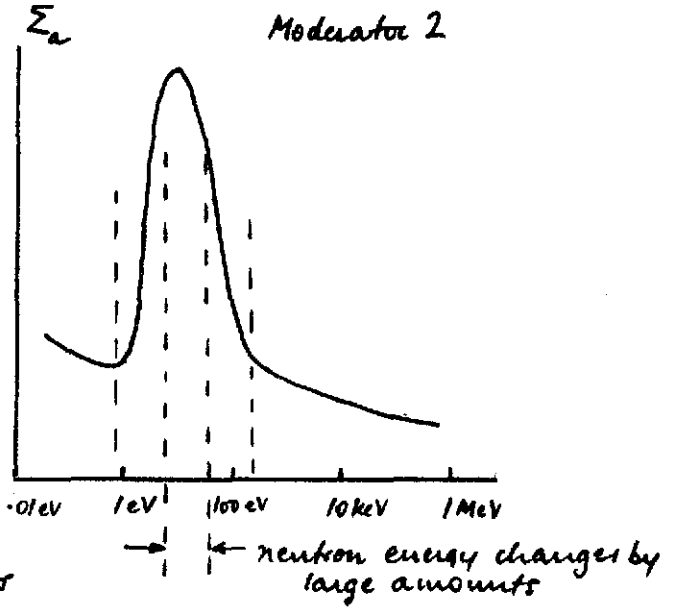


Fig. 2.

Effect of Moderator on Resonance Capture

Apart from this, during the slowing down process there will be less collisions and hence less neutron absorption by the moderator atoms themselves if we use moderator 2 rather than 1.

Slowing Down Mechanism

Having established that we want to slow the neutrons down in as few collisions as possible we shall now examine how this might be achieved.

There are two slowing down mechanisms:

(1) inelastic scattering (with fuel nuclei)
(2) elastic scattering (with moderator nuclei)

(Inelastic scattering with moderator nuclei is not possible because the energies are too low, and even with uranium nuclei it is only possible down to about 100 keV. In any case, it is relatively unimportant, as we shall see later. Elastic scattering with fuel nuclei may be ignored, because there the energy loss per collision is negligible).

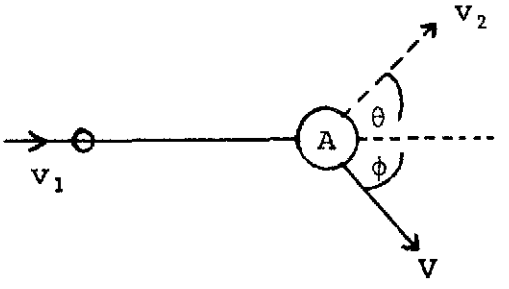


Fig. 3. Elastic Collision of Neutron with Moderator Nucleus

In an elastic collision the energy loss ΔE of a neutron of initial energy E is given by:

$$\frac{\Delta E}{E} = \frac{4A \cos^2\phi}{(A+1)^2} \qquad \text{(1)}$$

This expression shows that the fractional energy loss depends both on the incident angle ϕ and on the mass of the moderator nucleus. For many collisions the energy loss must be weighted according to the probability of a collision occuring at an angle ϕ, and a weighted mean energy loss per collision can then be found.

Now consider that N collisions with moderator nuclei are required to slow the fast neutrons (energy E_1) down to thermal values (energy E_{th}). The energy after the first impact is E_2, after the second E_3, and so on. We can therefore say that:

$$\frac{E_1}{E_{th}} = \left(\frac{E_1}{E_2}\right) \times \left(\frac{E_2}{E_3}\right) \cdots\cdots \left(\frac{E_i}{E_{i+1}}\right) \cdots\cdots \left(\frac{E_N}{E_{N+1}}\right) = \left(\overline{\frac{E_i}{E_{i+1}}}\right)^N$$

where $\overline{(E_i/E_{i+1})}$ signifies the average energy loss per collision.

Taking logs to base e gives

$$\ln\frac{E_1}{E_{th}} = N\,\overline{\log\frac{E_i}{E_{i+1}}} = N\xi \qquad (2)$$

ξ is called the *mean logarithmic energy decrement*, and is given by

$$\xi = 1 + \frac{(A-1)^2}{2A}\ln\left|\frac{A-1}{A+1}\right| \qquad (3)$$

(This expression can be obtained after some manipulation when the weighted mean of $\cos^2\phi$ is applied to equation (1). It is not my intent to bore you with the details since it is only calculus).

Substituting E_1=2MeV and E_{th}=0.025eV into (2) gives us

$$\ln\frac{2\times10^6}{0.025} = 18.2 = N\xi$$

$$\text{i.e.} \quad N = \frac{18.2}{\xi} \qquad (4)$$

This shows that a small number of slowing down collisions can only be achieved with a large value of ξ, or a small value of A since a fairly close approximation for ξ is

$$\xi \simeq \frac{2}{A+\frac{2}{3}} \qquad (5)$$

For A>10, this approximation is good to within 1%.

At this point it might be constructive to consider the effect of inelastic scatter in the fuel. If we assume that this reduces the average neutron energy from 2MeV to perhaps 1 MeV, then equation (4) changes to

$$N\xi = \ln\frac{1\times10^6}{0.025} = 17.5$$

This means that the number of collisions the neutrons have to make to reach thermal energies is reduced to $17.5/\xi$, ie, by less than 4%. For this reason, any slowing down effects by inelastic scatter in the fuel are usually ignored, and the number of collisions to thermalize is still taken as $18.2/\xi$.

If the moderator is not a single element, but a compound such as D_2O, the effective value of ξ is given by

$$\xi_{(D_2O)} = \frac{2\sigma_{s(D)}\xi_{(D)} + \sigma_{s(O)}\xi_O}{2\sigma_{s(D)} + \sigma_{s(O)}} \qquad (6)$$

Table 1 shows the accurate values of ξ (equation 3) of a number of light materials which might be suitable as moderators.

TABLE 1

Mean Logarithmic Decrements

	ξ	Collisions to Thermalize
H^1	1.000	18
H^2	0.725	25
He^4	0.425	43
Be^9	0.206	83
C^{12}	0.158	115
H_2O	0.927	20
D_2O	0.510	36
BeO	0.174	105

Slowing Down Power and Moderating Ratios

A small number of collisions to thermalize is obviously desirable, but this is of no use on its own unless the collisions actually occur. This implies that λ_s, the mean free path for scattering collisions, must be small. Therefore

$$\Sigma_s = \frac{1}{\lambda_s} = N'\sigma_s$$

must be large. This immediately rules out gases as moderators, because N' would be too small for the neutrons to be slowed down within a reasonable distance.

The overall effectiveness of a material for slowing down neutrons is measured by the product $\xi\Sigma_s$, which is known as the *Slowing Down Power*. You should be able to show that it is the average decrease in the log of neutron energy per cm of path.

λ = mean free path.

Table 2 shows the slowing down powers of the solid and liquid moderators introduced in Table 1. The value for helium is also shown to demonstrate the unsuitability of a gas.

TABLE 2

Slowing Down Powers and Moderating Ratios

	ξ	Σ_s (cm^{-1}) (a)	$\xi\Sigma_s$	Σ_a	$\xi\Sigma_s/\Sigma_a$
He (b)	0.425	21×10^{-6}	9×10^{-6}	? very small	? large
Be	0.206	0.74	0.15	1.17×10^{-3}	130
C (c)	0.158	0.38	0.06	0.38×10^{-3}	160
BeO	0.174	0.69	0.12	0.68×10^{-3}	180
H_2O	0.927	1.47	1.36	22×10^{-3}	60
(100%) D_2O	0.510	0.35	0.18	0.33×10^{-6} (d)	5500 (d)
D_2O	(reactor grade) 99.75%				

(a) Σ_s values of epithermal neutrons (ie, between ~1 and ~1000 eV)
(b) at S.T.P.
(c) reactor-grade graphite.
(d) 100% pure D_2O.

Not only must the moderator be effective in slowing down neutrons, but it must also have a small capture cross section. Neutrons are slowed down to decrease radiative captures compared to fission captures, and obviously the whole purpose of moderation would be defeated if the moderator nuclei themselves captured neutrons.

A reasonable indication of the overall quality of a moderator is the *Moderating Ratio*, which combines the slowing down power and the macroscopic capture cross section:-

$$\text{Moderating Ratio} = \frac{\xi\Sigma_s}{\Sigma_a} \qquad (7)$$

We are now in a position to draw some interesting conclusions from Table 2.

H_2O has excellent slowing down properties, and is often used as a fast neutron shield (neutrons must be slowed down before they can be absorbed. Why?) Unfortunately its Σ_a is too high to permit its use as a moderator for natural uranium fuel, and enrichment will be necessary.

Be, BeO and graphite have lower values of Σ_a, and can be used with natural uranium fuel provided it is in metal form. The use of natural uranium compounds with more attractive physical and chemical properties (such as UO_2 or UC) is not feasible with these moderators, because of the reduction in the concentration of uranium atoms. The British line of power reactors used a graphite moderator with natural uranium metal fuel (their earlier Magnox stations), and in the early '60s they changed to graphite with enriched UO_2 fuel (the AGR stations).

In the U.S., an abundance of U-235 and a tradition of using it in nuclear submarines led to all out development of light water reactors with relatively highly enriched fuel and a relatively poor moderator.

You can see from Table 2 that heavy water is by far the best moderator as far as its nuclear properties go, and of course its use was adopted for the CANDU line of reactors (CANDU = CANadian-Deuterium-Uranium). Its Σ_a is so low that natural uranium can even be used in compound form as UO_2.

The substance used as a moderator must be very pure. It is usually used, in a reactor, in larger amounts than any other material, eg, the volume of carbon in a graphite moderated reactor is 70 to 80 times that of the fuel. A very small amount of impurity in a moderator can substantially increase its capture cross section. The addition of 1 boron atom to every million graphite atoms would increase the capture cross-section of graphite by 25%.

For the same reason the isotopic purity of D_2O must be kept high. The addition of 0.25% H_2O to pure D_2O more than doubles the capture cross section. The isotopic purity of moderator D_2O is kept at 99.75% by weight or better. This is known as *reactor-grade D_2O*. This isotopic purity is not easy to maintain since heavy water is hygroscopic, nevertheless it is imperative that all H_2O be excluded. For example, a 0.25% decrease in isotopic purity in Pickering would cause a reduction in reactivity equivalent to a fuel burn-up loss of 21 full power days. This represents an amount of money which is so high as to be almost outside the audible range of the human ear.

The Diffusion of Neutrons Through the Moderator

Many parameters which determine the design of a reactor are dependent on the way neutrons are slowed down and diffuse in the moderator.

Neutrons diffuse through a material as a result of being scattered by nuclei. Neutrons virtually never collide with each other because the neutron density is so much smaller than the atomic density. The treatment of neutron diffusion, which is a process similar to the diffusion of electron in a metal, is too complicated to include in this course, and we shall therefore restrict ourselves to a pictorial representation.

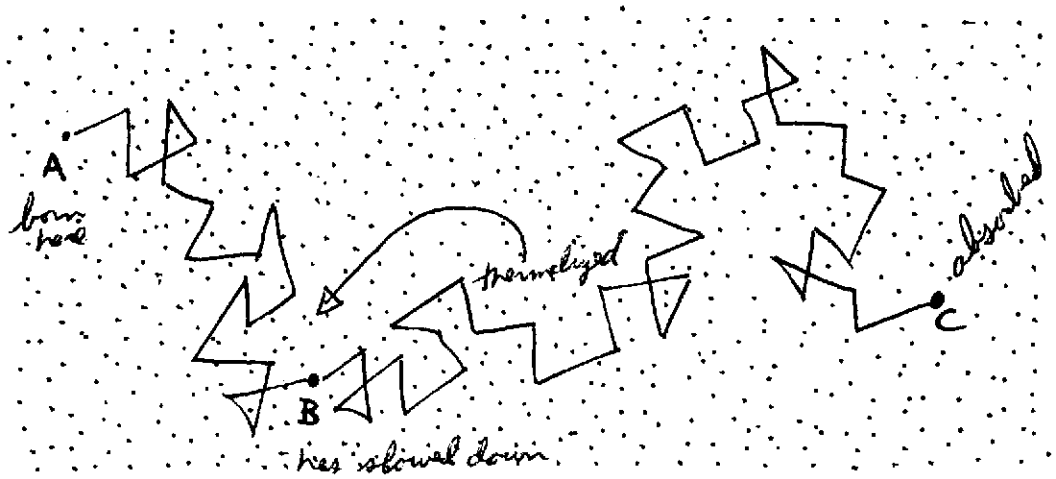


Fig. 4. Neutron Diffusion In A Moderator

A fission neutron born at A is thermalized in $18.2/\xi$ collision to arrive at B. The mean square of the "crow-flight" distance AB (it always seems to be assumed that crows bomb along in straight lines) is given by the expression

$$6L_s^2 = \overline{(AB)^2} \qquad (8)$$

L_s is called the *slowing down length*, and it is an important parameter in reactor physics. For D_2O L_s= 11 cm, and hence $\overline{(AB)^2}$ is about 725 cm^2, with $\overline{AB}$ around 23 cm*). This can be compared with an average total distance travelled during slowing down, which is the number of collisions times the mean free path for elastic scatter. For D_2O, this turns out to be around 100 cm.

*) You realise that $\overline{x}^2 \neq \overline{x^2}$, don't you?

After the neutron has been thermalized at B, it will zip around in the moderator before it is finally absorbed at C. A similar relationship again applies, namely

$$6L^2 = \overline{(BC)^2}, \qquad (9)$$

where L is the *diffusion length*. For D_2O this is very large at 100 cm, and you will probably realise that this reflects the exceptionally low Σ_a of D_2O.
(In fact, other things being equal, L^2 is inversely proportional to Σ_a.)

TABLE 3

Slowing Down and Diffusion Lengths

Moderator	L_s (cm)	L (cm)
H_2O	5.6	2.76
D_2O*	11.0	100
Be	9.2	21
Graphite	18.7	64.2

* reactor-grade

The slowing down length determines the optimum distance between adjacent fuel channels of a heterogenous reactor. This distance is called the *lattice pitch*, and the values of Table 3 should explain why H_2O moderated reactors are smaller than D_2O moderated reactors.

The low value of L for H_2O reflects its high Σ_a, and you can see that *overmoderating* a light water reactor (ie, having a greater lattice pitch than is needed) would carry such a severe penalty in terms of neutron absorption that it is never contemplated. On the other hand, overmoderating a heavy water reactor by a few cm makes very little difference to the absorption. In this case the designer might wish to increase the lattice pitch somewhat to give himself more flexibility for the design of end fittings and end shields. (All of the CANDU reactors are overmoderated - a pressure tube design makes it almost impossible to be otherwise.)

The values of L given in the table of course apply to a moderator alone, because it is assumed that the thermal neutrons are finally absorbed by moderator nuclei. In a reactor containing fuel, the large majority of thermal neutrons will be absorbed by fuel nuclei and the effective diffusion length is then considerably reduced. The slowing down length should not be affected much because the contribution of the fuel nuclei to the slowing down process is negligible. For the sake of comparison, the values for the Pickering cores are $L_s^2 = 145\ cm^2$ and $L^2 = 224\ cm^2$.

A final point is that the fraction of neutrons escaping from the core, called the *leakage*, also depends on L^2 and L_s^2. We shall leave the discussion of this to the ~~next~~ next 3rd lesson.

ASSIGNMENT

1. Why is ξ_{H_2O} so much closer to ξ_H than ξ_{D_2O} is to ξ_D?

2. How many collisions are required for neutrons to lose, on average, 99% of an initial energy of 2 MeV in D_2O? Compare this with the total number of collisions required to reach thermal energies.

3. Calculate the total zig-zag path made by fission neutrons slowing down to thermal energy in Helium gas. Compare this with the corresponding distance in H_2O.

4. Calculate the moderating ratio for reactor-grade D_2O, and insert this value in Table 2.

5. The time taken by a neutron to slow down from energy E_o to Energy E_t is given by

$$t_s = \frac{\sqrt{2m}}{\xi \Sigma_s}\left(\frac{1}{\sqrt{E_t}} - \frac{1}{\sqrt{E_o}}\right) \quad (m = \text{neutron mass})$$

   Calculate the slowing down times for H_2O, D_2O and graphite. Can you derive this expression?

6. For neutrons that have just been thermalized in reactor-grade D_2O, calculate the total zig-zag path, the number of collisions and the survival time before capture.

7. Explain why the values of L_s and L for the Pickering cores differ from those of D_2O.

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J.U. Burnham

Nuclear Theory - Course 127

EFFECT OF ENRICHMENT, FUEL ARRANGEMENT AND FUEL BURNUP ON THE FOUR FACTOR FORMULA

When considering the function and properties of the moderator, it was stated that a moderator has to be provided, in addition to fuel, in a reactor in order to slow down the neutrons to thermal energies.

A reactor containing fuel alone can not maintain a chain reaction unless the fuel is highly enriched and it operates as a fast reactor. However, it was not specified how the fuel and moderator would be arranged. There are two possible arrangements:

(a) A homogeneous system, in which the fuel and moderator are intimately mixed together. The uranium would either be in solution in the moderator or in a fine suspension called a slurry.

(b) A heterogeneous system, in which the fuel is in the form of lumps or rods arranged in a regular manner in the moderator.

The effect of such arrangements on the four factor formula, as well as the effect of enrichment and fuel burnup, will now be considered.

Homogeneous System

Suppose that the fuel and moderator are intimately mixed. When the fission neutron is born, it would be in contact with the moderator immediately and would collide with several moderator nuclei before encountering another fuel nucleus. The neutron would not therefore have enough energy to cause fast fissions in U-238 and $\epsilon = 1$. What of the other factors?

The number of neutrons, η , produced for each thermal neutron captured in the fuel will depend only on the fuel composition. If ν fast neutrons are produced at each U-235 fission, then:

$$\eta = \frac{\nu N_5 \sigma_f}{N_5\sigma_5 + N_8\sigma_8} = \frac{\nu \sigma_f R}{R\sigma_5 + \sigma_8} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where N_5 and N_8 are the numbers of U-235 and U-238 atoms, respectively, per unit volume of fuel, σ_f is the thermal neutron fission cross section of U-235, σ_5 and σ_8 are the absorption cross sections of U-235 and U-238, respectively, and R is the ratio N_5/N_8.

For natural uranium
$$R = 0.715\% = 0.00715$$
$$\nu = 2.48$$
$$\sigma_f = 580 \text{ barns}$$
$$\sigma_5 = 700 \text{ barns}$$
$$\sigma_8 = 2.8 \text{ barns}$$

$$\text{Hence } \eta = \frac{2.48 \times 580 \times 0.00715}{(0.00715 \times 700) + 2.8} = 1.32$$

$$\text{Now } k_\infty = \eta \epsilon pf = 1 \text{ for criticality}$$

$$\text{or } pf = \frac{1}{1.32} = 0.77$$

Thus, the product pf must be at least equal to 0.77 if such a homogeneous system is to be critical and a chain reaction be maintained. Is this possible with natural uranium fuel? If natural uranium fuel is to be used, the only way to vary p and f is to vary the ratio of moderator to fuel atoms in the homogeneous mixture. Table I below shows how p, f and the product pf vary as the ratio of moderator to fuel atoms is changed in a homogeneous mixture of graphite and natural uranium. The value of k_∞ is also given to show that it is less than 1, which is the minimum value, ignoring leakage, required for criticality.

TABLE I

nat. U + graphite

Mod. Atoms / Fuel Atoms	p	f	pf	k_∞
200	0.579	0.889	0.515	0.68
300	0.643	0.842	0.541	0.71
400	0.682	0.800	0.546	0.72
500	0.693	0.762	0.528	0.70

From the table it may be seen that the problem is that as the moderator/fuel ratio is increased, p increases, because there is better thermalization of neutrons, but f decreases, because there is more neutron capture in the moderator and less neutron capture in the fuel. The maximum value pf can have is

only 0.55, whereas the value must be 0.77 to sustain a chain reaction. So a chain reaction cannot be maintained with a homogeneous mixture of graphite and natural uranium.

A similar situation exists with homogeneous systems of natural uranium and either light water or beryllium. With heavy water as moderator, the optimum ratio of moderator to fuel gives a maximum value of pf of 0.78, which corresponds to a maximum value of k = 1.03. This does not allow for neutron leakage nor does it allow for buildup of fission product poison. This means that the size of reactor required, with a natural uranium - heavy water homogeneous mixture, would be too large to be practical.

The Effect of Fuel Enrichment

What can be done to make such a system practical? One answer lies in increasing the value of η. For a practical reactor, in which fuel consumption and poison accumulation are allowed for, k would have to be around 1.1 to 1.2. Therefore the value of η required is given by:

$$\eta = \frac{1.2}{pf} = \frac{1.2}{0.78} = 1.54 \text{ for } D_2O \text{ moderator}$$

$$\eta = \frac{1.2}{0.55} = 2.18 \text{ for graphite moderator}$$

The only way to increase η is to increase the U-235 concentration (R) in the fuel, ie, enrich the fuel. Fig. 1 shows how the value of η varies with the U-235 concentration in the fuel. The U-235 concentration in natural uranium is 0.00715 (0.715%). For a homogeneous D_2O moderated system to be practical, the U-235 concentration would have to be increased to about 0.02 in order to obtain a value of 1.54 for η. Thus the fuel would contain 2% U-235. It would appear from the graph that the required value of η of 2.18 for a graphite moderated system, cannot be attained. However, enrichment of fuel decreases the U-238 content and increases p. Thus, the value of η, required for a homogeneous

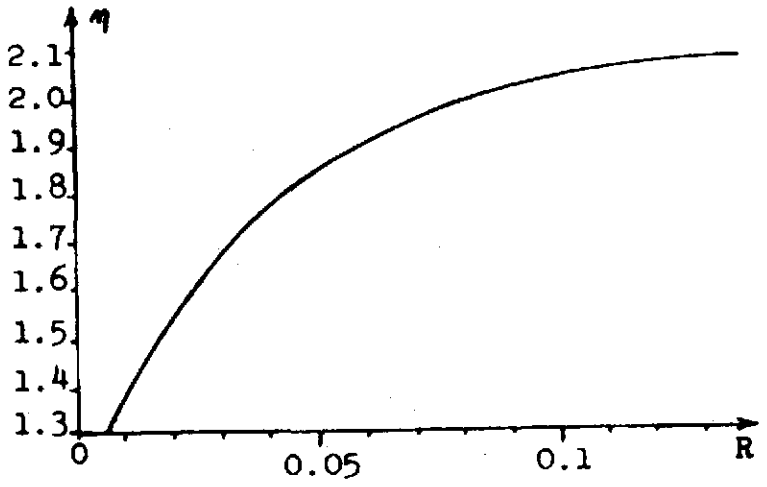


Fig. 1

mixture of graphite and fuel, decreases so that it can be attained with an enrichment of about 20% U-235.

It would also seem, from the graph, that there is little to be gained in the value of η by making R greater than 0.1. However, further enrichment does increase the value of p because of the decrease in the U-238 content.

Heterogeneous System

Even though enrichment does enable a homogeneous system to go critical, practical difficulties with such a system still exist. Refuelling problems, transportation of the heat released, fission product contamination, and criticality problems outside the reactor, are some of the difficulties encountered. Then, of course, there is also the very high capital cost of an enrichment plant and the obvious advantages to be gained using natural uranium, which requires only chemical processing. The alternative is to use a heterogeneous system if this is feasible. In such a system the fuel is located in discrete lumps in the moderator, as shown in Fig. 2, so that each fuel lump is surrounded by moderator material.

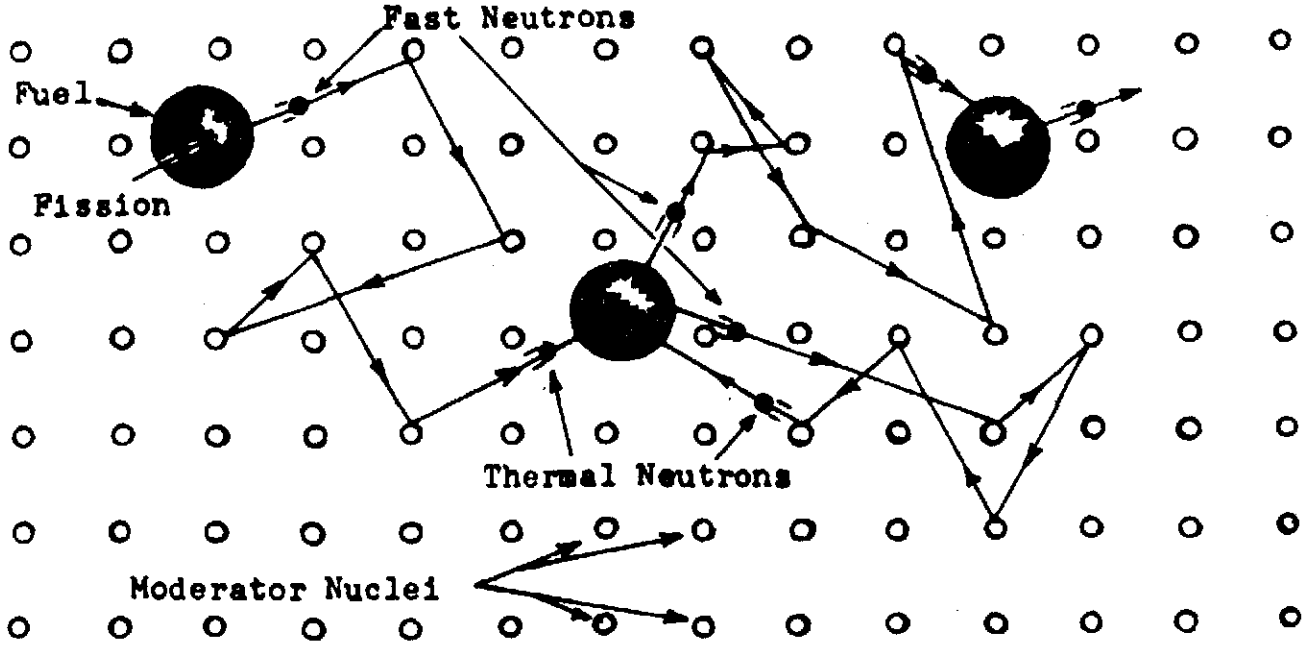


Fig. 2

Fissions then occur in the fuel lumps and the neutrons are partially slowed down by inelastic scattering as they escape

from the fuel. The slowing down process is completed by elastic scattering collisions with moderator nuclei before the neutrons enter other lumps of fuel. The separation between fuel lumps will determine how many elastic scattering collisions occur and, therefore, whether the neutron with average energy of 2 Mev is thermalized or whether neutrons of all energies are thermalized.

In practice, the fuel is in the form of rods or fuel ELEMENTS arranged in bundles, the rods being arranged in a regular pattern or LATTICE in the moderator, as shown in Fig. 3.

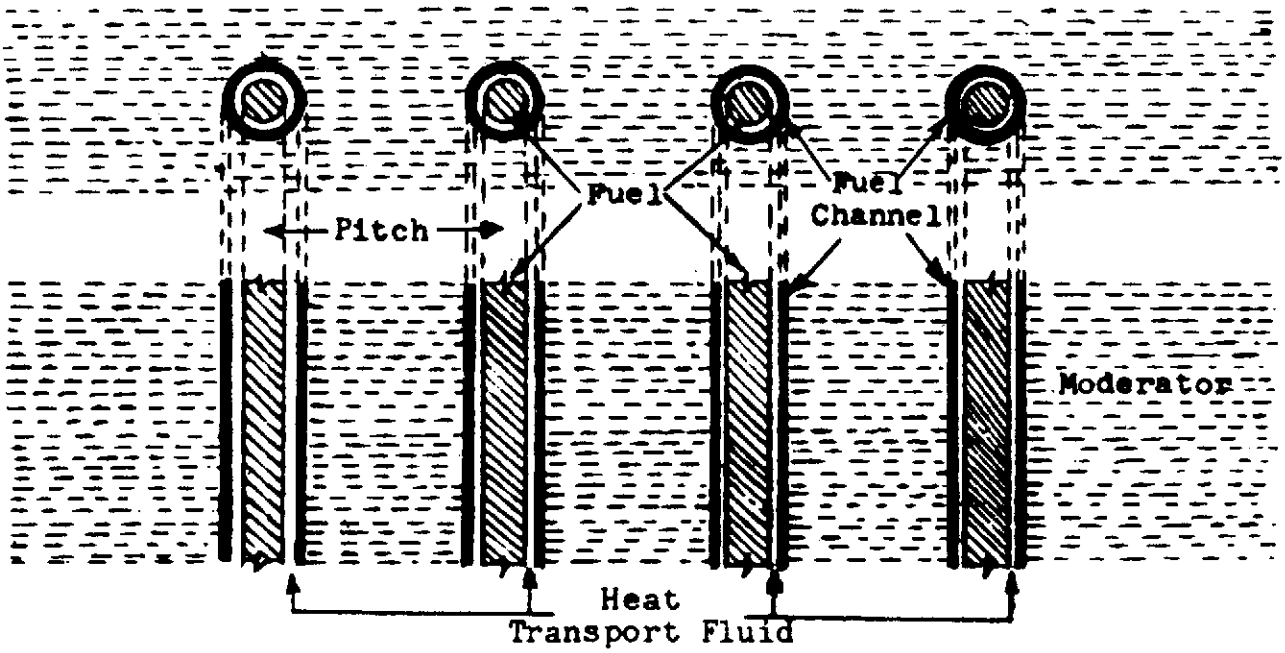


Fig. 3

This arrangement of rods inside tubes, or fuel channels, has several advantages:

(a) The location of the fuel is fixed by the fuel channel.

(b) Replacement of spent fuel by new fuel is easier.

(c) Heat can be removed from the fuel by passing the heat transport fluid along the fuel channel.

What nuclear advantages are to be gained by such an arrangement? The advantages, if any, will depend on what changes occur in the factors ϵ , η , p and f.

(a) The fast fission factor ϵ is increased. The fast fission neutrons are now released in the fuel rods and are, therefore, more likely to cause fast fissions in U-238 before

escaping from the rod into the moderator. The value of ϵ will be between 1.02 and 1.03 instead of 1.0.

(b) The value of p, the resonance escape probability, is increased. Some slowing down by inelastic scattering occurs in the fuel. Also, if the fuel rods are placed at a distance apart, comparable to the average distance travelled by the neutron while slowing down, then the neutron will stand a good chance of becoming thermalized before entering the next fuel rod. Values of p of around 0.9 can be obtained by such an arrangement, which will undoubtedly increase pf sufficiently to enable a chain reaction to be maintained without enrichment.

There is a limit to the lattice PITCH, or rod separation. If the pitch is increased so that all the neutrons are thermalized, the value of p increases but the neutron capture increases. If the separation or pitch is too great, so that the neutrons are thermalized well before they enter a fuel lump, the reactor is said to be overmoderated. If the rods are too close, the reactor is undermoderated. With a moderator such as heavy water, which has a low capture cross section, there is a tendency to overmoderate in order to make p as large as possible without substantially decreasing f.

(c) The thermal utilization factor f is decreased. In a heterogeneous system, the average thermal neutron flux in the fuel is lower than it is in the moderator. There is, therefore, a tendency for relatively fewer neutrons to be captured in the fuel than there is in a homogeneous system. However, the decrease in f is small and, with heavy water moderator, the value of f is still higher than 0.9.

(d) The value of η is unchanged by using a heterogeneous system.

The overall effect is to increase the value of k sufficiently to make the chain reaction possible, without enrichment. The maximum possible value of k_∞ is now greater than that given by:

$$k_\infty = 1.03 \times 0.9 \times 0.9 \times 1.32 = 1.102$$

Example:

For the NPD equilibrium core:

$\eta = 1.229$; $\epsilon = 1.021$; $p = 0.918$; $f = 0.933$

Therefore $k_\infty = 1.075$

For Douglas Point:

η = 1.1757; ϵ = 1.0267; p = 0.8995; f = 0.9503

Therefore k_∞ = 1.032

There are some significant differences between the two sets of parameters:

(a) The Douglas Point reactor is larger than the NPD reactor and a smaller allowance is necessary for neutron leakage. Hence, the value of k_∞ is smaller for Douglas Point than for NPD.

(b) The lattice pitch for Douglas Point is only 9" compared with 10-1/4" for NPD. This accounts for the smaller value of p and the larger value of f for the Douglas Point reactor.

(c) The values are given for fuel from which U-235 has been removed by fission and in which some buildup of Pu-239 has occurred. The difference in the values of η and ϵ are thus due to a difference in burnup of the fuel in NPD and Douglas Point.

The degree of fuel subdivision will also have some effect on ϵ. The factor ϵ will increase with rod radius, since the longer the fission neutron remains in the fuel the more likely fast fissions are to occur. The fact that all the Douglas Point fuel is 19-element fuel, whereas the NPD fuel is a mixture of 7-element and 19-element fuel, will tend to make ϵ for NPD higher than for Douglas Point. However, this is completely masked by the effect of fuel burnup.

Effect of Fuel Burnup

As U-235 fissions occur in the fuel, the U-235 is being used up. However, Plutonium-239 is being produced from U-238 and Pu-241 is being produced by neutron capture in Pu-239 and then in Pu-240. Both Pu-239 and Pu-241 are fissionable with thermal neutrons. So, initially at least, we have fissionable U-235 burnt and fissionable Pu-239 and Pu-241 being produced. Later Pu-239 and Pu-241 will be burnt as well as produced and, eventually, all the U-235 will be used up and Plutonium alone will be burnt.

The burnup of U-235 and the production of Plutonium does not affect either p or ϵ to any great extent. However, the capture and fission cross sections of Pu-239 are substantially greater than those of U-235. This means that, although the

Plutonium is not produced as fast as U-235 is burnt up, there is initially an increase in the product ηf and, therefore, in the multiplication factor k. However, later, the burnup of Plutonium causes k to start to decrease even if poisons are ignored. Before the change in k with burnup can be shown, a method must be established of measuring fuel burnup. The possible definitions of burnup are as follows:

(a) Burnup is the percentage of the original fissile atoms burnt.

(b) Burnup is the percentage of the total fuel atoms burnt.

(c) Burnup is the heat extracted (in Megawatt days) per tonne (10^6 gms) of fuel.

Burnup, then, may be measured as a percentage of U-235 atoms burnt. If all the U-235 is used up, the burnup is 100%. Since Plutonium is being produced and burnt as well as all the U-235, the burnup can be greater than 100%.

If the U-235 forms a fraction E of the total fuel atoms,

$$b\% \text{ of U-235 burnup} = Eb\% \text{ total fuel burnup} = 10{,}000\ Eb \text{ Megawatt-days/tonne}$$

Therefore, if a reactor core was big enough to give a burnup of 10,000 Mwd/tonne, then Eb = 1 or 1% of all fuel atoms are burnt. If E = 0.00715, as it is in natural uranium,

$$b = \frac{1}{0.00715} = 140\% \text{ of U-235 atoms}$$

That is, the total number of U-235, Pu-239 and Pu-241 atoms fissioned is equal to 140% of the original amount of U-235 present.

A typical curve, showing how k varies with burnup, is shown in Fig. 4. No allowance has been made for fission product poison buildup which, initially, tends to mask out the Plutonium buildup. The increase in reactivity, during the early fuel irradiation, does occur, however, and an allowance must be made for it.

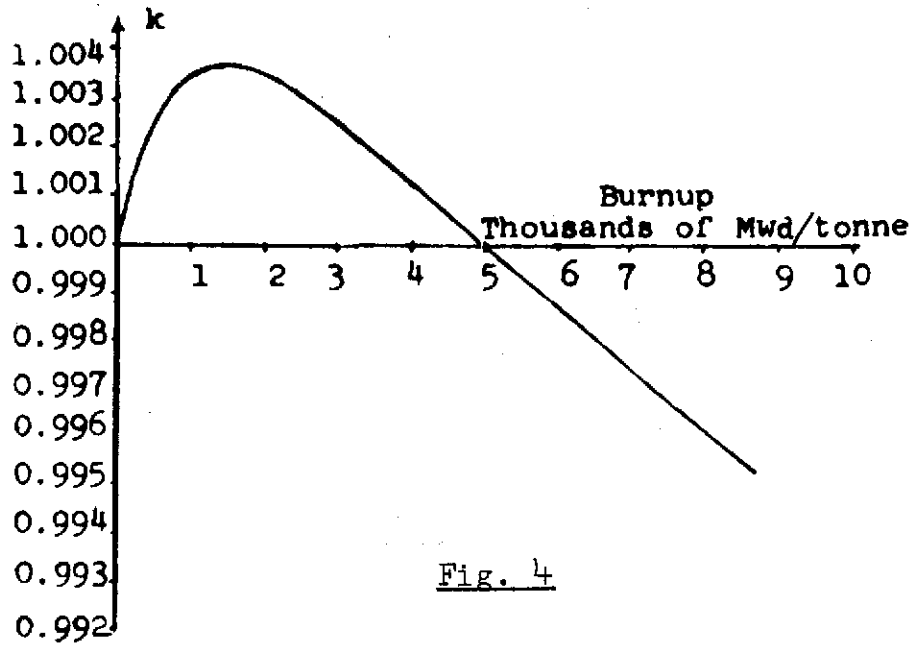


Fig. 4

ASSIGNMENT

1. Explain, in terms of the four factor formula, why a chain reaction can not be maintained by a reactor containing a homogeneous mixture of moderator and natural uranium.

2. What factors in the four factor formula are changed when the fuel is enriched and how do the changes enable criticality to be achieved?

3. How are the factors in the four factor formula changed when a heterogeneous system of moderator and fuel is used instead of a homogeneous system?

4. What advantages are to be gained by using fuel rods, in channels, rather than lumps of fuel arranged in the moderator?

5. (a) Give one definition of burnup.

   (b) What percentage of U-235 atoms are burnt if the burnup of natural uranium is 6500 Megawatt-days/tonne?

   (c) Why does the multiplication factor increase during initial fuel burnup and then decrease later?

A. Williams

Nuclear Theory - Course 127

FLUX DISTRIBUTION AND CRITICAL SIZE

Discussions so far have centred around the reduction of neutron losses, due to radiative capture, so as to increase some, or all, of the factors in the four factor formula and, thus, maintain a chain reaction. Neutrons are also lost through escape or leakage out of the reactor and consideration will now be given to neutron conservation by reduction of these losses. Neutron leakage out of the reactor may be reduced in one of two ways:

(a) By increasing the size of the reactor.

(b) By using a reflector.

The first of these two methods will now be discussed.

The multiplication factor, k_∞ , for a reactor of infinite size is connected with η , ϵ , p and f by the four factor formula:

$$k_\infty = \eta \epsilon p f \quad \text{.....(1)}$$

If there is no leakage of neutrons out of the system, the condition for criticality is

$$k_\infty = 1$$

When neutron leakage has to be considered, the multiplication factor, k_e, (known as k - effective), allows for this leakage as follows:

$$k_e = k_\infty - \text{neutron leakage} \quad \text{.....(2)}$$

It may be shown that:

$$k_e = k_\infty - M^2B^2 \quad \text{.....(3)}$$

where M is the neutron migration length, which depends on the core composition, and B^2 is the buckling, which depends on the size and shape of the reactor and on the flux distribution. For instance in a cylindrical reactor:

$$B^2 = \frac{(2.405)^2}{R_e^2} + \frac{\pi^2}{L_e^2} \quad \text{.....(4)}$$

where R_e is known as the EXTRAPOLATED radius and L_e the extrapolated length. R_e and L_e are slightly greater than the physical radius and length respectively. These quantities will be more clearly defined later in the lesson.

From the expression for B^2, (equation (4)), it is clear that the value of B^2 and, therefore, the neutron leakage will decrease if R or L increase.

Critical Size of a Reactor

It was stated above that neutron leakage from a reactor can be decreased by increasing the dimensions of the reactor. This is also evident from simple geometric considerations. As the volume of the reactor increases, the rate of fissioning increases. However, the surface area also increases and so, therefore, does the neutron leakage. When the dimensions of a reactor are increased, the volume increases faster than the surface area. Hence, the number of neutrons produced in the reactor increases faster than the neutron leakage and there is a net increase in the production of neutrons.

When the size of the reactor is such that the number of neutrons produced is exactly the same as the neutrons removed by fission, radiative capture and leakage, the chain reaction is maintained. The reactor is then critical, (with $k_e = 1$), and the size of the reactor is, then, its CRITICAL SIZE. The critical size will, therefore, be reached when neutron leakage has been reduced sufficiently for $k_e = 1$.

The critical size referred to is the minimum critical size with new fuel. Any size less than the minimum critical size cannot go critical. However, a reactor of this size would not remain critical since the quantity of fuel is being reduced by fissions and fission products, which absorb neutrons, accumulate. Thus, the reactor size is usually substantially greater than its minimum critical size.

A spherical reactor will have the smallest critical size because a sphere will have the smallest surface area for a given volume. A cylindrical reactor will have a bigger critical size that a spherical one and a critical cubical reactor will be bigger still. A spherical reactor would be somewhat difficult to build and, therefore, reactor of cylindrical shapes are usually chosen.

Neutron Flux Distribution

The thermal neutron flux, ϕ, in a reactor, is the quantity that determines the number of fissions that take place per second. The higher the neutron flux, at any point in a reactor, the higher the rate of fissioning at that point and the greater the power produced at that point.

The neutron flux may be defined in a number of ways:

(a) If ϕ is the neutron flux at a point the number of neutron induced reactions per cc per second at that point will be given by:

$$\text{No. of reactions/cc/sec} = \phi N \sigma$$

where N is the number of reacting nuclei per cc at that point and σ the cross-section for that reaction.

If σ_f is the fission cross-section then $\phi N \sigma_f$ will be the number of fissions/cc/sec.

(b) The thermal neutron flux, at a point in a reactor, is the product of the thermal neutron density, n, at that point and the average speed of the thermal neutrons.

ie, $\phi = nv$

(c) The thermal neutron flux, at a point in a reactor, is the number of thermal neutrons per second, travelling in all possible directions, which cross a unit area placed at that point.

Definitions (a) and (b) are more mathematically correct and more acceptable than (c). However, definition (c) sometimes helps to give some physical significance to neutron flux. The unit used for neutron flux is usually neutrons/cm^2/sec.

The thermal neutron flux need not be the same at every point in a reactor. In fact, the flux is usually a maximum at the centre and has a distribution across a reactor which is characteristic of the shape of the reactor, provided that nothing is done to alter this natural distribution.

In a cylindrical reactor, shown in Fig. 1, there are two directions in which the flux distribution is considered. These directions are along the axis, Oz, and along the radial direction, Or from the centre of the reactor.

The axial flux distribution is given by:

$$\phi = \phi_m \cos\left\{\frac{\pi z}{L_e}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

That is to say, the flux at a distance z from O, along Oz in either direction, is given by the above equation, ϕ_m being the maximum flux at O.

The flux at any distance, r, from O is given by:

$$\phi = \phi_m J_o\left\{\frac{2.405r}{R_e}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

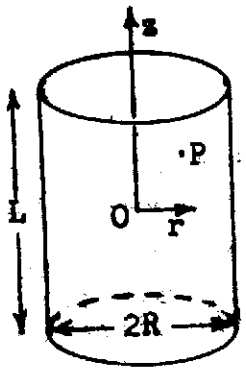


Fig. 1

where ϕ_m is again the maximum flux at O and J_o is a special function known as a zero order Bessel function. However, the radial flux distribution is given, very closely by a simpler cosine formula:

$$\phi = \phi_m \cos\left\{\frac{\pi r}{2R_e}\right\} \quad \text{.....(7)}$$

The quantities R_e and L_e are the extrapolated radius and length which were mentioned earlier in the lesson.

Fig. 2 shows both these cosine distributions graphically and also shows the significance of R_e and L_e.

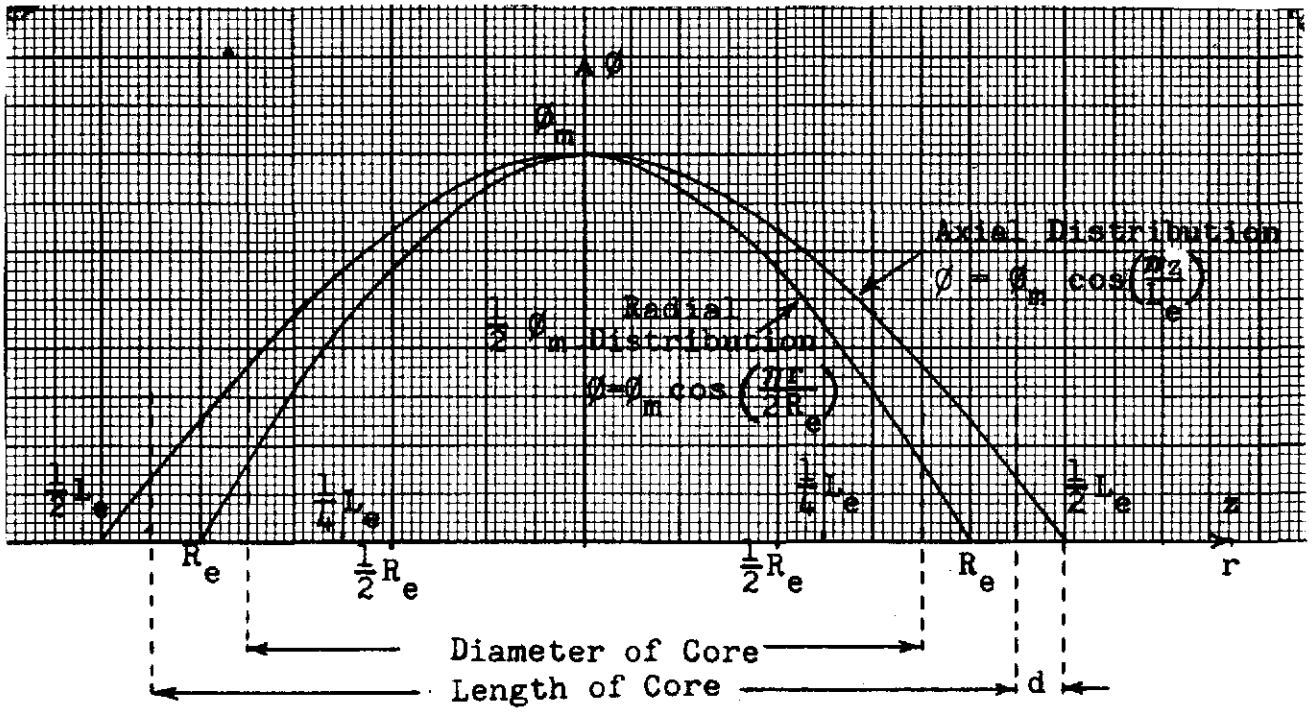


Fig. 2

It may be seen that the flux falls off to zero at some distance, d, outside the physical boundary of the core. This distance, d, is known as the EXTRAPOLATION DISTANCE. Thus if R and L are the actual physical dimensions of the core:

$$R_e = R + d$$

and $$L_e = L + 2d$$

The flux at any point P (Fig. 1), in the reactor, with axial coordinate z and radial coordinate r would be given by:

$$\phi = \phi_m \cos\left\{\frac{\pi r}{2R_e}\right\} \cos\left\{\frac{\pi z}{L_e}\right\} \quad \text{.....(8)}$$

Effect of Neutron Flux Distribution on Reactor Power

An examination of the flux distribution graphs will show that:

(a) The thermal neutron flux has a definite maximum value at the centre but falls to nearly zero at the edge of the reactor. Since the rate of fissioning and, consequently, the power depends on $\phi\Sigma_f$, the maximum power is being produced at the centre and very little power is being produced from the fuel in the outer regions of the core.

(b) The total power produced by the reactor depends on the average thermal neutron flux, the relationship between the average flux, ϕ_a, and the power P (in Megawatts), being:

$$\phi_a = 3 \times 10^{12} \frac{P}{U} \quad \text{.....(9)}$$

where U is the total weight of uranium fuel in the reactor, in tonnes.

Thus, the power required and the average flux determine the total fuel loading and the core size. With the flux distributions, shown in the cylindrical case, ϕ_a is only 27.5% of the maximum flux, ϕ_m, ie, the maximum flux is 3.6 times greater than the average.

The average flux can only be increased, in a bare reactor, by increasing the maximum flux. However, the maximum flux is usually limited by the maximum fuel heat rating or by the severity of the Xenon transient. The maximum fuel rating will be reached much sooner in the centre of the reactor and so there is very poor utilization of the rest of the fuel.

Example

In NPD $\phi_m = 8 \times 10^{13}$ n/cm^2/sec

Hence $\phi_a = 2.2 \times 10^{13}$ n/cm^2/sec

Therefore $U = \dfrac{3 \times 10^{12} \times 80}{2.2 \times 10^{13}} = \underline{10.9 \text{ tonnes}}$

In Douglas Point the radial flux distribution is deliberately flattened over a portion of the reactor to increase ϕ_a.

ASSIGNMENT

1. (a) How is the effective multiplication factor connected with the neutron leakage?

   (b) On what two factors does the neutron leakage depend?

2. (a) Why does relative neutron leakage decrease as the reactor dimensions increase?

   (b) What is meant by the critical size of a reactor?

   (c) What reactor shape would have the smallest critical size and why is this so?

3. Define thermal neutron flux and specify the units in which it is measured.

4. (a) Write down the equations giving the approximate radial and axial flux distributions in a cylindrical reactor.

   (b) What two disadvantages result from this type of flux distribution?

A. Williams

Nuclear Theory - Course 127

THE FUNCTION AND PROPERTIES OF THE REFLECTOR

Reduction of neutron leakage by increasing the reactor core size has already been considered. The core size is increased until neutron losses are balanced by neutron production. However, the resulting flux distribution leaves a lot to be desired, particularly from the point of view of fuel utilization.

Neutron leakage from the core can also be reduced by surrounding the core with a substance which scatters or reflects neutrons back into the core. Such a substance is known as a REFLECTOR. Such a reflector also has desirable effects on the flux distribution.

The Function of the Reflector

Fig. 1 below shows the function of a reflector diagrammatically. Fig. 1(a) shows a "bare" core with many neutrons escaping. In Fig. 1(b) a substance has been placed around the core to reflect most of the neutrons back into the core.

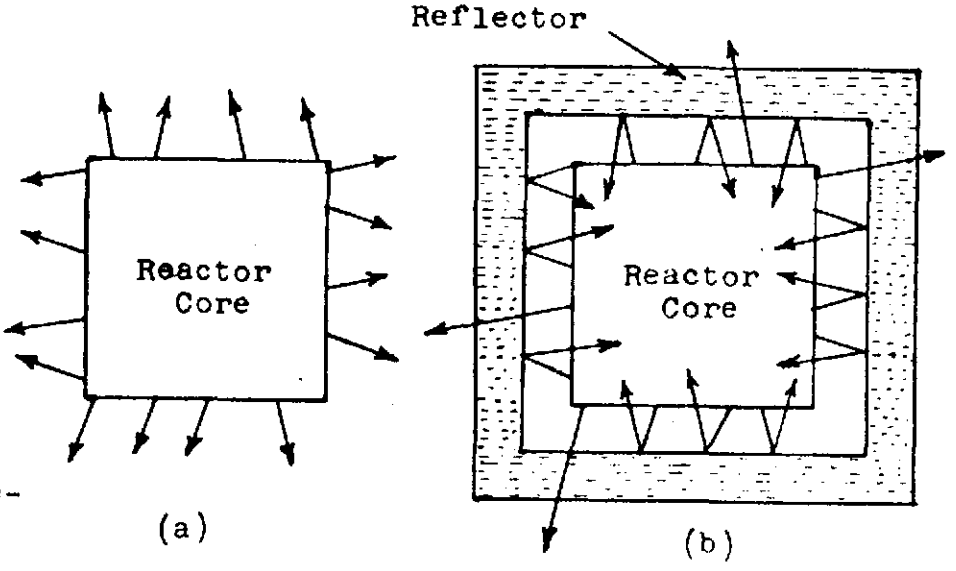


Fig. 1

It is evident that, with the reflector, more neutrons are available for fission because the leakage is smaller. Therefore, the core size does not have to be increased as much in order for the reactor to go critical. That is, the critical size of a reflected core is smaller than that of a bare core. Alternatively, if the size of the core is kept the same, higher fuel burnups can be achieved with consequent reduction in fuel costs.

Reflector Properties

The neutrons are reflected back into the core by scattering collisions between the neutrons and the reflector nuclei. The efficiency of a substance as a reflector is measured by a quantity known as the REFLECTOR COEFFICIENT (β).

The reflector coefficient may be defined as the fraction of the neutrons entering the reflector which are reflected back into the core.

For a reflector in the form of a slab, the reflector coefficient for thermal neutrons is connected with the diffusion length L by the equation:

$$\beta = 1 - \frac{4D}{L} \quad \text{.....(1)}$$

where D is a quantity known as the DIFFUSION COEFFICIENT which depends on the scattering and absorption cross sections of the material.

Although Equation (1) only applies to slab geometry, it can be used to illustrate that the greater the value of L and the smaller the value of D the closer β becomes to unity, ie, the more efficient the material is as a reflector. The values of D and L and the maximum value of β that could be obtained with an infinitely thick reflector are tabulated below for H_2O, D_2O, graphite and beryllium.

TABLE I

Material	L (cm)	D (cm)	β for an Infinite Slab	β for a Slab Thickness 2L
H_2O	2.76	0.17	0.780	0.772
D_2O	100	0.88	0.966	0.965
Beryllium	21	0.54	0.901	0.900
Graphite	64	0.94	0.944	0.940

The desirable properties of a thermal neutron reflector material may be summarized as follows:

1. Thermal neutrons are reflected by elastic scattering between them and reflector nuclei. Therefore, the macroscopic elastic scattering cross section, Σ_s, should be

high. If this is the case, the value of D will be low. Thus the density and the value of σ_s, must be high. Table I shows that H_2O has the lowest value of D.

2. The capture or absorption cross section, σ_a, should be low so that as few neutrons as possible are lost by capture. This requirement ensures a high value of L. However, since the escaping neutrons would have been lost, in any case, without the reflector, a low value of σ_a is not quite as important as it is in a moderator. Hence, H_2O is quite acceptable as a reflector for a natural uranium-fuelled reactor, even though it is not as efficient as D_2O. A light water reflector is also an excellent fast neutron shield and its use might well help to avoid the use of a thermal shield.

These desirable properties are, of course, almost identical with those expected in a moderator. The best thermal neutron reflectors are those materials that make the best moderators because they have small σ_a / σ_s ratios.

In the case of fast neutrons the most effective scattering mechanism is inelastic scattering. The best fast neutron reflectors are, therefore, the heavier materials. Hence, uranium or thorium are the materials used as reflectors in fast reactors such as the Enrico-Fermi reactor. Such a reflector is frequently referred to as a BLANKET since production of fissile material from fertile material occurs in the reflector.

How thick should a reflector be? The reflector coefficient β increases, initially, as the reflector thickness increases. However, as shown in Fig. 2, very little is to be gained by increasing the thickness beyond a value equal to 2L. Table I also shows that the value of β, for a reflector thickness of 2L, is within 1% or less of the maximum possible value. Since the desirable reflector properties are so similar to those of the moderator, it would be an

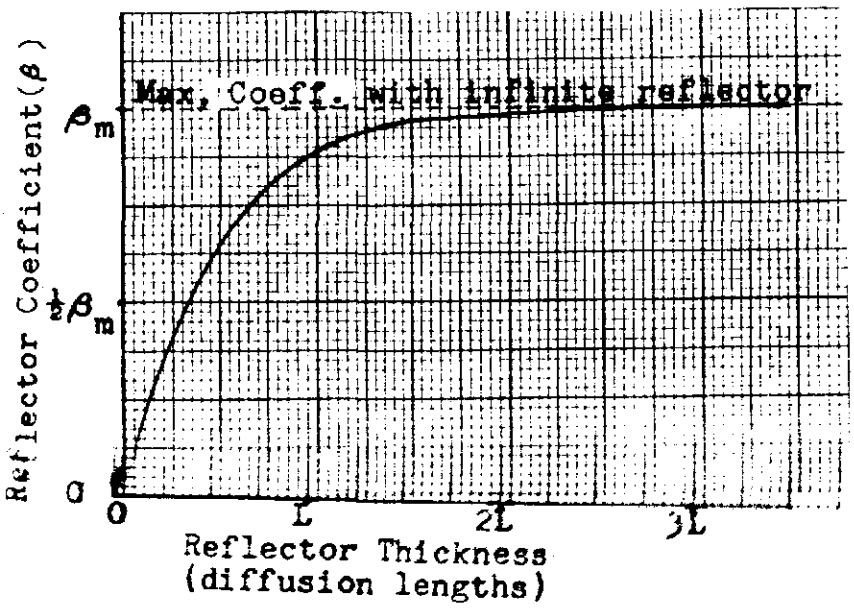


Fig. 2

advantage for the reflector to be merely an extension of the moderator so that both can be enclosed in the same reactor vessel.

With H_2O a thickness of 2L is only 5.5 cm, whereas for D_2O it is 200 cm. It would require a considerable amount of D_2O to extend the moderator by 200 cm, since this additional D_2O is being placed on the outside of the core.

Effects of Adding a Reflector

The effects of placing a reflector around the core can be summarized as follows:

1. The neutron flux distribution is "flattened", ie, the ratio of the average flux to the maximum flux is increased. This is illustrated in Fig. 3 for a reactor in which all the neutrons are assumed to have the same energy (eg, a fast reactor). Fig. 4 shows the change in flux distribution in a thermal reactor. The hump in the reflected curve in Fig. 4 is due to the fact that fast neutrons escape into the reflector and become thermalized in the reflector.

   The flux at the edge of the core is now over 50% of the maximum instead of only 20% in the bare core. When both the radial and axial flux distributions are considered, in a reactor like NPD, the average flux is increased from 27.5% to 42% of the maximum flux.

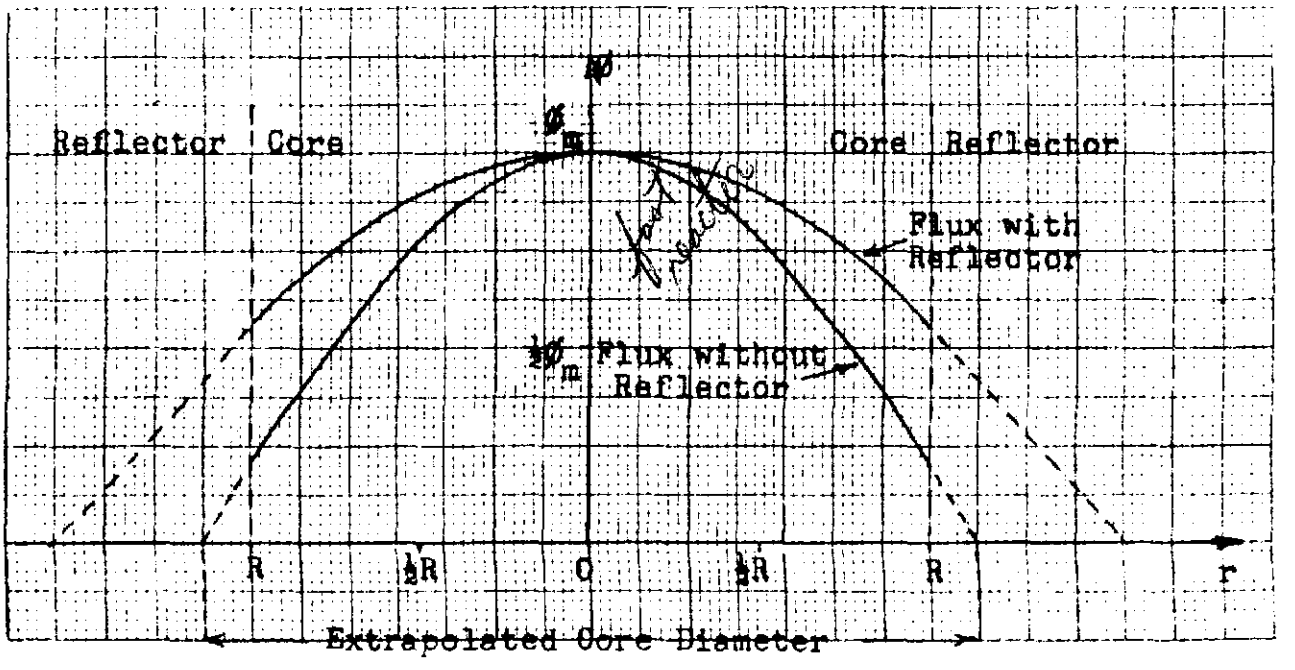


Fig. 3

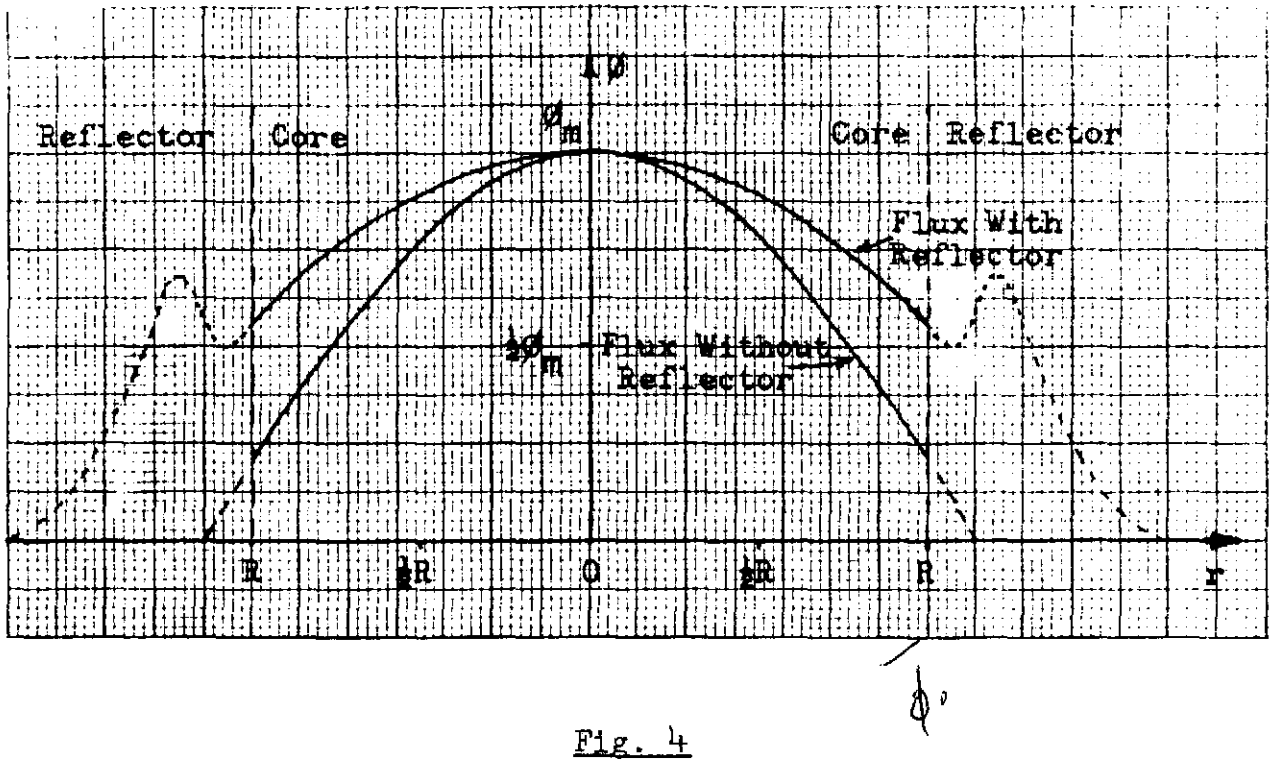


Fig. 4

2. Because of the higher flux at the edge of the core, there is much better utilization of fuel in the outer regions. This fuel, in the outer regions of the core, now contributes much more to the total power production.

3. The neutrons reflected back into the core are now available for fission. This means that the minimum critical size of the reactor is reduced. Alternatively, if the core size is maintained, the reflector makes additional reactivity available for higher fuel burnup.

   The decrease in the critical size of core required is known as the REFLECTOR SAVINGS.

4. The average flux, ϕ_a, in NPD is now given by:

$$\phi_a = 0.42 \times 8 \times 10^{13} = 3.32 \times 10^{13} \text{ n/cm}^2\text{/sec}$$

Therefore, for the same weight of uranium (10.9 tonnes) used in the previous lesson, the power P that can be produced is given by:

$$P = \frac{\phi_a U}{3 \times 10^{12}} = \frac{3.32 \times 10^{13} \times 10.9}{3 \times 10^{12}} = \underline{120} \text{ Mw}$$

This compares with 80 Mw, which can be produced with the same fuel in the bare core.

Alternatively, if the power output is kept the same, the weight of uranium required is given by

$$U = \frac{3 \times 10^{12} \times 80}{3.32 \times 10^{12}} = \underline{\underline{7.2}} \text{ tonnes}$$

This is a 30% decrease in the amount of fuel required.

ASSIGNMENT

1. (a) What is the basic purpose of a reflector?

   (b) What quantity is used to determine its efficiency?

2. (a) State two essential properties of a thermal neutron reflector.

   (b) Give two reasons why light water is suitable as a reflector but not as a moderator.

3. Why are the heavier materials the best fast neutron reflectors?

4. Why are reflector thicknesses never greater than twice the diffusion length of the reflector material?

5. (a) How does a reflector affect thermal neutron flux distribution?

   (b) State two important consequences of this change in flux distribution.

A. Williams

Nuclear Theory - Course 127

REVIEW OF TERMS

So far in this course, consideration has been given to a reactor which is only just critical and the conditions under which this criticality can be achieved. The discussions have been centred around the conditions under which a chain reaction can just be maintained and the contributions made to these conditions by the moderator, reflector and fuel arrangement. No account has been taken of the changes which take place, in a reactor, when it is operated.

For the balance of this Reactor Theory course consideration will be given to disturbances of the steady state, ie, to factors that affect the neutron balance in a reactor and the way in which they are affected. The steady state of a reactor can be disturbed by a variety of causes, which fall into the following general categories:

(a) Power changes brought about by means of the regulating or protective system.

(b) Temperature changes.

(c) Formation of voids.

(d) Build-up of fission product poisons.

(e) Changes in the content of the fuel.

Many of these changes are time dependent, which means that they vary with time in some definite manner.

This first lesson, on this section, will review the terms which will be used in the following lessons.

Neutron Density

The thermal power, (in watts or joules/sec or Btu/min or any other appropriate unit), which is produced in a reactor is directly proportional to rate of fissioning or the fissions occurring per second. Each watt of power produced requires 3.1×10^{10} fissions/sec.

If the power produced is to remain constant, the rate of fissioning must remain constant and, consequently, the total

number of neutrons in the reactor must remain constant. The total number of neutrons in the reactor is referred to as the neutron population.

The number of neutrons per unit volume of the reactor is known as the neutron density. Thus, neutron density is measured in neutron/cc.

If the reactor power is steady, the average neutron density remains constant. However, the neutron density may vary from one point to another in the reactor, having its maximum value at the centre and having the same distribution as the thermal neutron flux.

Neutron Power

The thermal power in a reactor is measured from the temperatures and heat transport flow rates, ie, from the thermal energy transported from the core. However, when changes occur in the rate of fissioning, there is some delay before the temperatures settle down. Thermal power changes lag behind changes in neutron density. It is, therefore, desirable that the neutron densities be measured so that a quicker indication, of changes occurring, can be obtained.

This measurement of neutron density is made, at one point in the reactor, with ion chambers and associated equipment. The electronic equipment is usually calibrated, in percent of full power, by comparison with thermal power. This measurement of neutron density is known as NEUTRON POWER. It is an instantaneous measurement.

It must be remembered that the changes in neutron power that will be considered, are not, necessarily, the same as the changes in thermal power that result from them.

Neutron Flux

The neutron flux at a point in a reactor has been defined as the product of the neutron density (n) and the average neutron velocity (v).

ie, $\phi = nv$ ....(1)

Alternatively, the rate at which a particular neutron reaction occurs may be used to define the flux from:

Number of reactions/cc/sec = $\phi\Sigma$ ....(2)

where ϕ is the neutron flux and Σ the macroscopic cross section for that particular reaction.

Thus, the higher the neutron flux, at a point in a reactor, the higher the rate of fissioning, the higher the power produced and the higher the neutron density at that point.

Criticality

Criticality is the reactor state attained when a chain reaction is just being maintained and the power remains constant, ie, the reactor is just critical.

When neutron losses are reduced, so that more neutrons are available for fission than are required just to maintain the chain reaction, then there is neutron multiplication. The power increases and the reactor is said to be supercritical.

When not enough neutrons are available for fission and the chain reaction cannot be maintained, then the power decreases and the reactor is subcritical.

The Multiplication Factor

The multiplication factor, k, is defined as the ratio of the number of neutrons in any one generation to the number of neutrons in the immediately preceding generation.

There are two multiplication factors. The multiplication factor k_∞ is the multiplication factor assuming no leakage, ie, the multiplication factor of a reactor system of infinite size. k_∞ is given by the four factor formula:

$$k_\infty = \eta \epsilon p f \quad \text{.....(3)}$$

A more practical multiplication factor is k-effective or k_e, which is the multiplication factor of the actual reactor, allowing for leakage.

$$k_e = k_\infty - \text{leakage} \quad \text{.....(4)}$$

$$k_e = k_\infty - M^2 \left\{ \frac{(2.405)^2}{R_e^2} + \frac{\pi^2}{L_e^2} \right\} \text{for a cylindrical reactor}$$

of extrapolated radius R_e and extrapolated length L_e.

Hence the product $\frac{M^2(2.405)^2}{R_e^2}$ represents the radial leakage

and the product $\frac{M^2\pi^2}{L_e^2}$ represents the axial leakage out of the reactor.

Reactivity

The quantity $(k_e - 1)$ is a measure of how far a reactor is from being just critical. It is, therefore, a quantity which is more significant than k_e itself, particularly where reactor regulation is involved. The REACTIVITY of a reactor is also a measure of how far the reactor is from being just critical. The reactivity, denoted by δk or ρ, is defined in some texts by the equation:

$$\delta k = \rho = \frac{k_e - 1}{k_e} \quad \text{.....(5)}$$

and is therefore the fractional deviation of k_e from unity.

However k_e is always close to unity and:

$$\delta k \approx k_e - 1 \quad \text{.....(6)}$$

The reactivity, as defined by equation (5), may have to be used for some calculation but, within Ontario Hydro Operations Division, the reactivity will normally be taken as $k_e - 1$, unless otherwise specified.

When the reactor is just critical $\delta k = 0$
When the reactor is supercritical δk is positive
When the reactor is subcritical δk is negative

Reactivity is normally measured in MILLI-K or one-thousandth of k (0.001 k).

Thus if $k = 1.007$, $\delta k = 7$ milli-k (7 mk).

The reactor is then said to be 7 mk over critical or to have 7 mk positive reactivity.

When $k = 0.995$, $\delta k = -5$ mk

The reactor is then 5 mk subcritical and has 5 mk of negative reactivity.

Neutron Lifetime

The average time between successive neutron generations, in a reactor, is defined as the NEUTRON LIFETIME, $\mathcal{L}$.

The neutron lifetime of prompt neutrons is determined by the time for fissioning, the slowing down time of the fast neutrons and the time taken by the thermalized neutron to be captured (ie, its diffusion time). The lifetime is almost entirely determined by the diffusion time and is about 10^{-3} sec in D_2O moderated natural uranium reactors.

The delayed neutron lifetime is determined by the half-life of its precursor or the fission product from which it originates. Delayed neutron lifetimes vary from 0.07 sec to 80 sec.

Reactor Period

The simplest definition of reactor period is the time required for a reactor to change its neutron density or its power by a factor e (the exponential e = 2.716).

When the reactor is operating at a fixed power level, the period is infinite. A reactor has a finite, measurable period only when its power is changing.

The reactor instrumentation does not measure period directly, but rather the inverse of the period (1/T). It will be shown later that the inverse of the period is equal to the rate of change of the logarithm of neutron power, a quantity which is called the RATE LOG.

ie, rate log $= \frac{d\ (\log\ P)}{dt} = \frac{1}{T}$

Process Times

Several processes, that go on in a reactor, have already been considered and others mentioned. Each process has its own characteristic rate or time interval which are significant in the present series of lessons. The following table lists the approximate characteristic times of these processes in a D_2O moderated reactor.

	Process	Approx. Characteristic Times
(a)	Fission	10^{-14} sec
(b)	Slowing Down	10^{-4} sec
(c)	Diffusion of thermal neutrons	10^{-3} sec
(d)	Delayed neutron lifetimes	0.07 sec to 80 sec
(e)	Photoneutron lifetimes	1 sec to 2 weeks
(f)	Xenon poison (growth)	10 to 11 hours
(g)	Xenon poison (decay)	14 to 20 hours
(h)	Time constants for thermal effects	few seconds to many minutes
(i)	Fission product accumulation	lifetime of the fuel

3./ Criticality – chain reaction is just being maintained + power level is constant
critical – $\delta k = 0$
supercritical – δk +ve
subcritical – δk -ve

127.20-1

ASSIGNMENT

1. Define neutron density. no. of neutrons / unit volume n/cm^3.

2. (a) Explain the difference between thermal and neutron power. measure $\Delta T \cdot F$ (Δ temp × flowrate) of neutron density

   (b) What is the advantage of measuring neutron power over thermal power? neutron power measurements are instantaneous T.P. & N.P. thermal power changes need time to adjust

3. Define the term "criticality" and explain the terms "critical", "supercritical" and "subcritical".

4. (a) Define the neutron multiplication factor, k. $\frac{\#\text{ neutrons in one gen}}{\#\text{ neutrons in preceding gen}}$

   (b) In terms of k_e, when is the reactor

   (1) just critical? $k_e = 0$

   (2) supercritical? k_e +ve

   (3) subcritical? k_e -ve

5. (a) Define the term "reactivity" and state the units in which it is measured. fractional deviation of k_e from unity milli-k.

   (b) When $k_e = 1.0075$, calculate the reactivity. $k_e = 1.0075$ $\delta k = k_e - 1.0 = 1.0075 - 1.000$

   (c) In terms of reactivity, when is the reactor $\delta k = .0075$ or 7.5 mk

   (1) just critical? $\delta k = 0$ 7.5 mk of positive reactivity.

   (2) supercritical? δk +ve

   (3) subcritical? δk -ve.

6. (a) Define "neutron lifetime". $\mathcal{L}$ – average time between successive neutron generations.

   (b) What is the main factor that determines the lifetime of prompt neutrons and what is the approximate value of this lifetime? the diffusion time (time for thermalized

   (c) What determines the lifetime of delayed neutrons and what range of values do they have? neutron to be captured

7. (a) Define "reactor period". time required for a reactor to change its neutron density or power by a factor e.

   (b) What is the quantity that is normally measured instead of the period and how is it related to the period?

$$\text{rate log} = \frac{d \log_e P}{dt} = \frac{1}{T}$$

A. Williams

Nuclear Theory - Course 127

LOW POWER CONSIDERATIONS

At low power Xenon poison and fission product buildup are of little significance. Thermal effect may also be ignored. Hence considerations of reactor operation at low power are somewhat simpler and more straightforward than those of high power operation.

Change of Neutron Flux and Neutron Power Following a Reactivity Change

Consider a reactor operating in a steady state with $k_e = 1$ but with no source of neutrons present. The reactivity is now changed by δk and then held fixed. This is known as a STEP CHANGE in reactivity. If n is the neutron density in the reactor, δk is the excess of neutrons from one generation to the next, ie, if the neutron density is n in one generation it will be increased by $n\delta k$ by the next generation. Since the generation time is ℓ, the rate of increase of neutrons is $\frac{n.\delta k}{\ell}$

ie, Rate of increase $= \frac{dn}{dt} = \frac{n.\delta k}{\ell}$

Rearranging we get $\frac{dn}{n} = \frac{\delta k}{\ell} . dt$

Integrating both sides $\int \frac{dn}{n} = \frac{\delta k}{\ell} \int dt$

$$\text{or } \log_e n = \frac{\delta k}{\ell} . t + \text{constant}$$

To determine the constant, let $n = n_o$ when $t = o$, ie, immediately prior to the reactivity change.

Then constant $= \log_e n_o$

$$\text{Therefore} \qquad \log_e n = \frac{\delta k t}{\ell} + \log_e n_o$$

$$\text{or} \qquad \log_e \left(\frac{n}{n_o} \right) = \frac{\delta k t}{\ell}$$

$$\text{or} \qquad \frac{n}{n_o} = e^{\frac{\delta k}{\ell} . t}$$

Hence $n = n_o e^{\frac{\delta k}{\ell} \cdot t}$ .....(1)

This equation gives the neutron density, n, at some time t sec following the δk change in reactivity.

The neutron flux and the power level, in a reactor, are both proportional to the neutron density and, therefore, both will change, with time, in the same way.

Thus $\phi = \phi_o e^{\frac{\delta k}{\ell} \cdot t}$ .....(2)

and $P = P_o e^{\frac{\delta k}{\ell} \cdot t}$ .....(3)

δk may be positive or negative in these equations and so the power will increase exponentially if δk is positive and decrease exponentially if δk is negative. The greater the value of δk the faster the increase or decrease in power will be. The smaller the value of the neutron lifetime, ℓ , the faster the increase or decrease in power will be. Therefore, the value of ℓ is important.

The Reactor Period

The reactor period was defined, in the previous lesson, as being the time required for the power (or flux or neutron density) to change by a factor of e. The significance of this definition is more apparent if equation (3) is examined.

$P = eP_o$ when $t = T$

So $T = \frac{\ell}{\delta k}$ is the reactor period.

The preceding equations now become:

$$P = P_o e^{\frac{t}{T}} \quad ; \quad \phi = \phi_o e^{\frac{t}{T}} \quad ; \quad n = n_o e^{\frac{t}{T}}$$

From these equations, it may be seen that the shorter the reactor period, ie, the smaller the value of T, the faster the power changes will be.

Power Changes With Prompt Neutrons Only

If all the neutrons in a reactor were prompt neutrons, the neutron lifetime, ℓ , would be about 0.001 sec.

Hence, if δk = 0.5 mk, $T = \frac{0.001}{0.0005}$ = 2 sec and, in 1 sec the power would increase by a factor of 1.65, ie, $P = 1.65\ P_o$.

If δk = 2 mk, $T = \frac{1}{2}$ sec and, in one second, $P = 7.4\ P_o$.

If δk = 5 mk, $T = \frac{1}{5}$ sec and, in one second, $P = 148\ P_o$.

The above examples indicate how rapid the power increases would be, even for small increases in reactivity. The graphs in Fig. 1 show the variation of power with time for various positive values of δk.

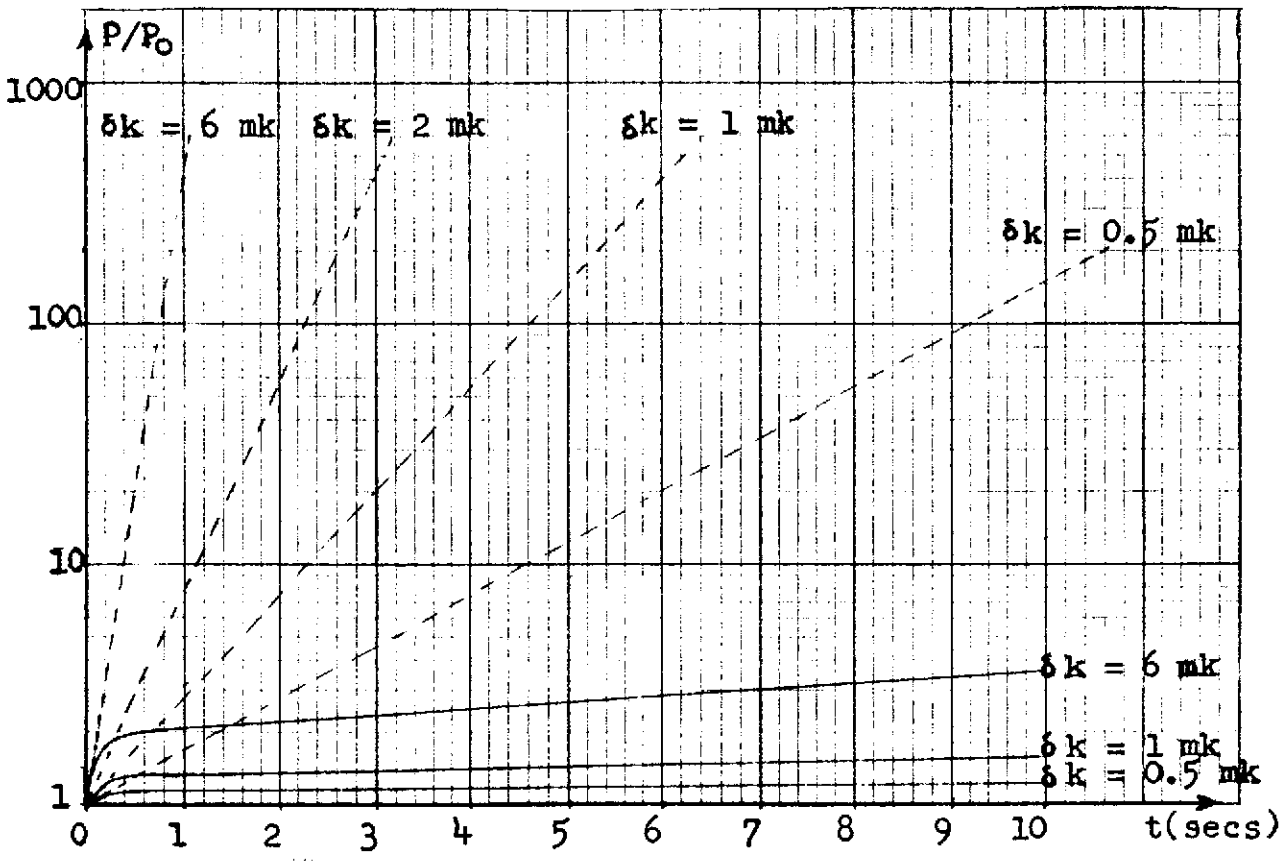


Fig. 1

The increases of power, with prompt neutrons only, are shown as dotted lines. With rapid power increases of the types shown, effective reactor regulation is not possible. Effective protection also becomes difficult since the fastest protective system will take at least 1 second to act. In this period of time, severe damage would have resulted from the high power level reached.

Fig. 2 shows the manner in which the power would decrease, for negative reactivities, if all neutrons were prompt neutrons.

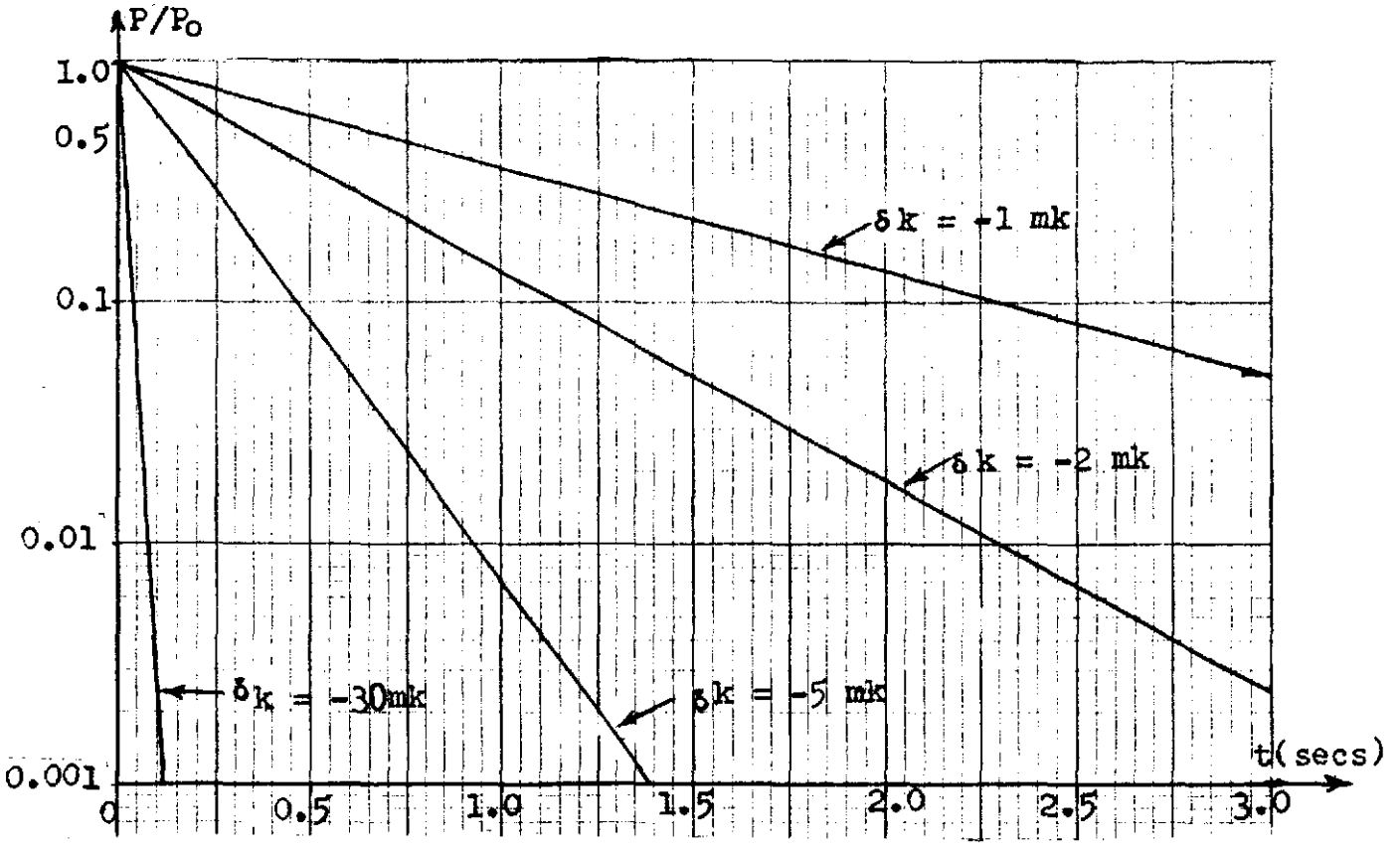


Fig. 2

The decrease in power is quite rapid, even for small negative reactivities. Negative reactivities, well in excess of -30 mk, can normally be introduced by the protective system and, with these large negative reactivities, the power would decrease to one-thousandth of full power in 0.2 sec or less.

Effect of Delayed Neutrons on Power Changes

The following table lists the fission yields, half-life and average life of the six delayed neutron groups:

Yield (%)	Half life (sec)	Average life (sec)	Yield x av. life
0.025	55.60	80.20	2.00
0.166	22.00	31.70	5.26
0.213	4.51	6.51	1.39
0.241	1.52	2.19	0.53
0.085	0.43	0.62	0.05
0.025	0.05	0.07	0.00
99.245	Prompt	0.00	0.00
100.000			9.23

The product of the yield and average life is determined for each group and the sum total of this column found (9.23).

The average life of all neutrons, both prompt and delayed, is this total divided by the total yield of 100%. The average lifetime is then this average life plus the diffusion time.

ie, Average lifetime = $\frac{9.23}{100}$ + 0.001 = 0.0933 sec.

Thus, although the delayed neutrons form only 0.755% of all the neutrons which result from fission, they change the average lifetime of all neutrons from 0.001 sec to approximately 0.1 sec, ie, by a factor of 100. Using this new value for ℓ :

when δk = 0.5 mk, T = 200 sec and, in 1 sec, P = 1.005 P_o
ie, only an increase in power of 0.5% in 1 second.

if δk = 2 mk, T = 50 sec and, in 1 sec, P = 1.02 P_o
if δk = 6 mk, T = 16.7 sec and, in 1 sec, P = 1.06 P_o

Therefore, the power increases are much less rapid than with prompt neutrons alone. However, it must be remembered that it would take a tenth to a fifth of a second for the delayed neutrons to become effective. During this initial fraction of a second, the power would increase in the manner described with prompt neutrons alone. The continuous graphs, in Fig. 1, show how the power increases both during this initial period and after the effect of the delayed neutrons are felt. As may be seen, the power increases do not become excessive during the response times of the regulating and protective system. So regulation and protection become practical realities.

Fig. 3 shows how the power decreases for negative reactivities when the effect of the delayed neutrons is considered. Initially the decrease in power is still due, entirely, to the decrease in prompt neutrons and the slopes of the graphs are, initially, equal to those of the corresponding graphs in Fig. 2 (shown in dotted line). After this initial rapid decrease in power, the delayed neutrons become the deciding factor. Each delayed neutron group, in turn, predominates, starting with the shortest half-life group. Eventually, the power decrease is governed entirely by the delayed neutron group having a 55.6 sec half-life.

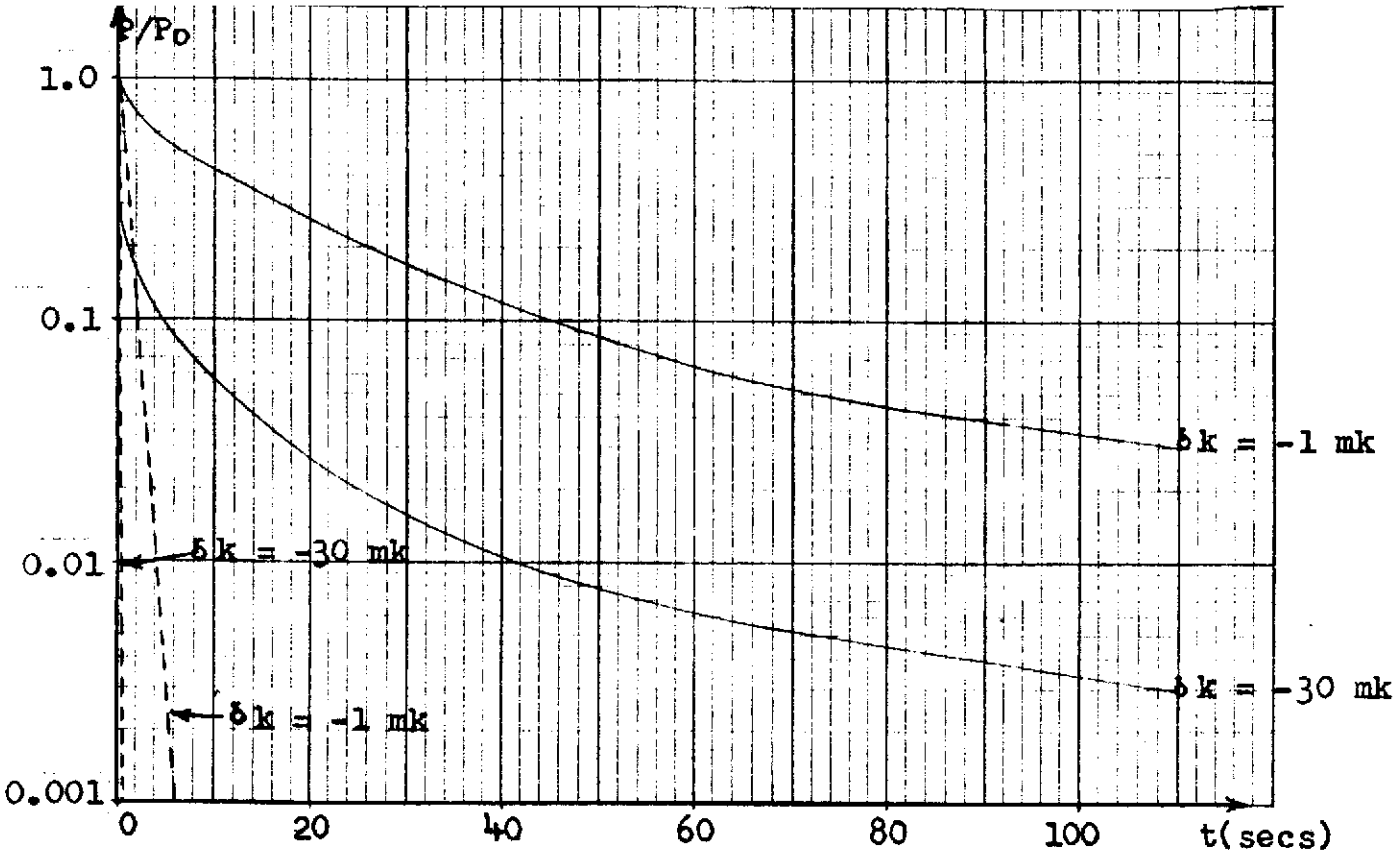


Fig. 3

It is evident that the delayed neutrons cause a considerable decrease in the rate of power reduction. For rapid shutdown, a substantial amount of negative reactivity is required to cause an initial rapid decrease in power, before the delayed neutrons slow down the power reduction. When $\delta k = -30$ mk the power drops to one-tenth of its value in 4 sec but it takes 40 sec for it to drop to 1/100th of its value and 30 min to drop 10 decades, eg, from 100 Megawatts to one-hundredth of a watt.

Prompt Critical

As the value of δk becomes more and more positive, the initial rapid rise in power gets longer and longer and the response of the regulating system has to become progressively faster.

When the reactivity is equal, in value, to the total delayed neutron yield from fission, then the chain reaction can be sustained by prompt neutrons, without the aid of delayed neutrons. The reactor is then said to be PROMPT CRITICAL.

The total delayed neutron yield = 0.755% = 0.00755

Therefore, the reactor is prompt critical when $\delta k = 0.00755 = 7.55$ mk.

By the time the reactivity has reached this value of 7.55 mk the initial rapid rise in power is substantial and the final steady reactor period is less that 2 seconds. The net result is an increase in power by a factor of 3 to 4 during the first second. Such increases are to be avoided and, so, the regulating system is usually designed to avoid δk values even approaching 7.55 mk. The period is normally kept above 10 sec and, preferably around 25 sec or higher.

In a thermal reactor the prompt critical condition is not something suddenly arrived at as the reactivity becomes equal to the delayed neutron yield. As shown in Fig. 4, in the curve for $\ell = 10^{-3}$ sec, there is no sudden decrease in reactor period at prompt critical. Therefore, it is not implied that a thermal reactor with $\delta k = 7.55$ mk is much more difficult to control than one in which $\delta k = 7.4$ mk, say.

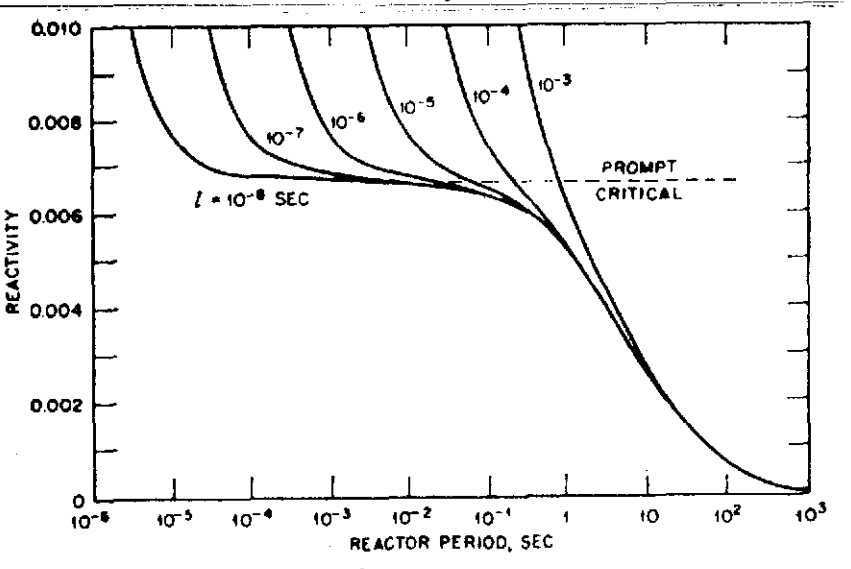


Fig. 4

However, in a fast reactor in which ℓ will have a much smaller value, there is a large decrease in reactor period at prompt critical. Thus the prompt critical condition is far more significant.

Effect of Neutron Sources on Power Changes

The equations at the beginning of the lesson were derived assuming that there was no source of neutrons, in the reactor, other than fission. A source of neutrons may be introduced into the reactor for the initial start-up. Alternatively, if D_2O is used as a moderator, photoneutrons may be produced by the absorption of fission product gamma rays in deuterium, according to the equation:

if $\gamma > 2$ meV

$$ {}_1H^2 + \gamma = {}_1H^1 + {}_0n^1 $$

These neutron sources may persist after the reactor has been shut down and the prompt and delayed neutrons have died down.

If the source strength is S neutrons/cc/sec, the equation for the neutron density variation is modified to:

$$ n = n_o e^{\frac{t}{T}} - \frac{0.001S}{\delta k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4) $$

If P_s is the source strength expressed in watts then the reactor power is given by:

doesn't ~~only~~ hold if $\delta k \approx 0$ or very small.

$$ P = P_o e^{\frac{t}{T}} - \frac{P_s}{\delta k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5) $$

From both these equations, it may be seen that, when δk is positive, the exponential term grows so fast that the source term is insignificant, ie, P_s is usually so small that it does not contribute to the total power produced. It should be noted that this equation does not apply for very small values of δk.

However, if δk is negative, the exponential term decreases and, when the delayed neutrons have died down sufficiently, the source term becomes significant. The power will eventually level out at $P = \frac{-P_s}{\delta k}$, as shown in Fig. 5, instead of continuing to follow the delayed neutron decay, as shown by the dotted line.

When $P_s = 30$ watts and $\delta k = -30$ mk; $P = \frac{30}{.03} = 1000$ watts and if $P_o = 100$ Mwatts; $\frac{P}{P_o} = 10^{-5}$

One big advantage of a photoneutron source is that it reduces the range of power which the neutron power instruments have to measure. With a photoneutron source, the range to be covered is around 6 decades, whereas otherwise it would be several decades more. There are logarithmic instruments that will cover 6 decades on one scale and linear instruments that will cover 8 decades fairly adequately with range switching but this is the limit of neutron power measurements.

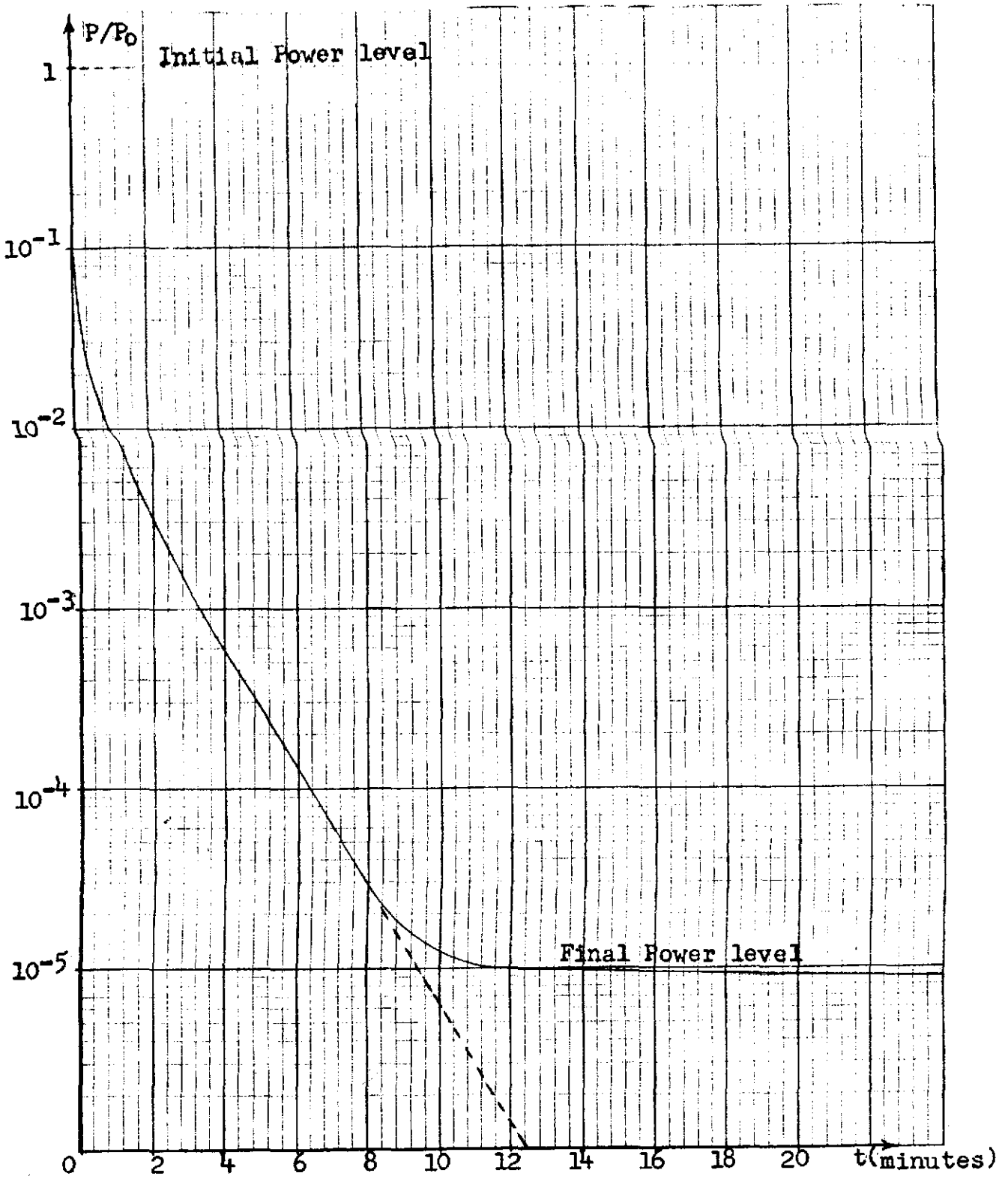

P/P0
Initial Power level
1
10-1
10-2
10-3
10-4
10-5
Final Power level
0
2
4
6
8
10
12
14
16
18
20
t(minutes)

Fig. 5

ASSIGNMENT

1. If all the neutrons, in a reactor, were prompt neutrons with a lifetime of 0.001 sec:

   (a) Calculate the reactor period and the power increase that occurs in 1 sec if $\delta k = 6$ mk.

   (b) What conclusions can be drawn, regarding reactor regulations and protection, with such power increases?

2. Assuming that, with delayed neutrons, the average neutron lifetime is increased from 0.001 sec to 0.1 sec:

   (a) Calculate the reactor period and the power increase that occurs in 1 sec if $\delta k = 6$ mk.

   (b) Comment on the significance of the answers when compared with the values obtained in 1(b).

3. Why is a larger amount of negative reactivity required, for a rapid shutdown, because all the neutrons are not prompt neutrons?

4. Explain what is meant by "Prompt Critical" and why this condition, or anything approaching it, should be avoided.

5. How does a neutron source in a reactor affect:

   (a) the power production?

   (b) the neutron power decrease after shutdown?

   (c) the neutron power instrumentation required for normal operation?

A. Williams

Nuclear Theory - Course 127

EFFECTS DUE TO TEMPERATURE CHANGES AND VOID FORMATION

When a reactor is operated at power, fission products and xenon poison build up and fissile material, in the form of U-235 and plutonium is consumed. These effects will be considered later. In addition to such effects, operation at power also causes increases in the temperatures of the fuel, heat transport fluid and moderator. It is the changes in reactivity, resulting from these temperature changes, that will be considered in this lesson.

Formation of voids in the heat transport fluid or moderator, due to boiling or accidental loss of fluid, is closely connected with temperature changes. Therefore, void formation and its effect on reactivity will also be discussed.

Effects Due to Temperature Change

In 1949, the NRX reactor at AECL was allowed to "run away". NRX is a heavy water moderated reactor which also uses control rods. The heavy water level was set 3 cm above the low power critical height and the control rods withdrawn. The reactor power was allowed to increase unchecked and the manner in which the power increased is rather unexpected. Fig. 1 shows the manner in which reactor power increased and the way in which the heavy water temperature changed.

The power initially increased exponentially as would be expected with a period of 33 sec. However, it does not continue to increase indefinitely as would have been expected. The reactivity started to decrease, because the temperature of the fuel rods increased. This causes the power increase to slow down. Later the reactivity decreased at a faster rate because of the increase in the heavy water temperature. The decrease in reactivity was sufficient to cause the reactor reactivity to become negative, with the result that the power reached a maximum value and then started to decrease. Thus the reactor is self-regulating with the temperature increases, preventing the power from continuing to rise, as it would otherwise have done. Of course, the initial period of 33 sec shows that the excess reactivity was only 3 mk in the first place. Had more reactivity than this been added, ie, had the moderator level been higher, it is quite possible that the power would have continued to rise. This example is not being used to demonstrate that reactor power would never increase continuously, but to show that there was a loss of reactivity due to

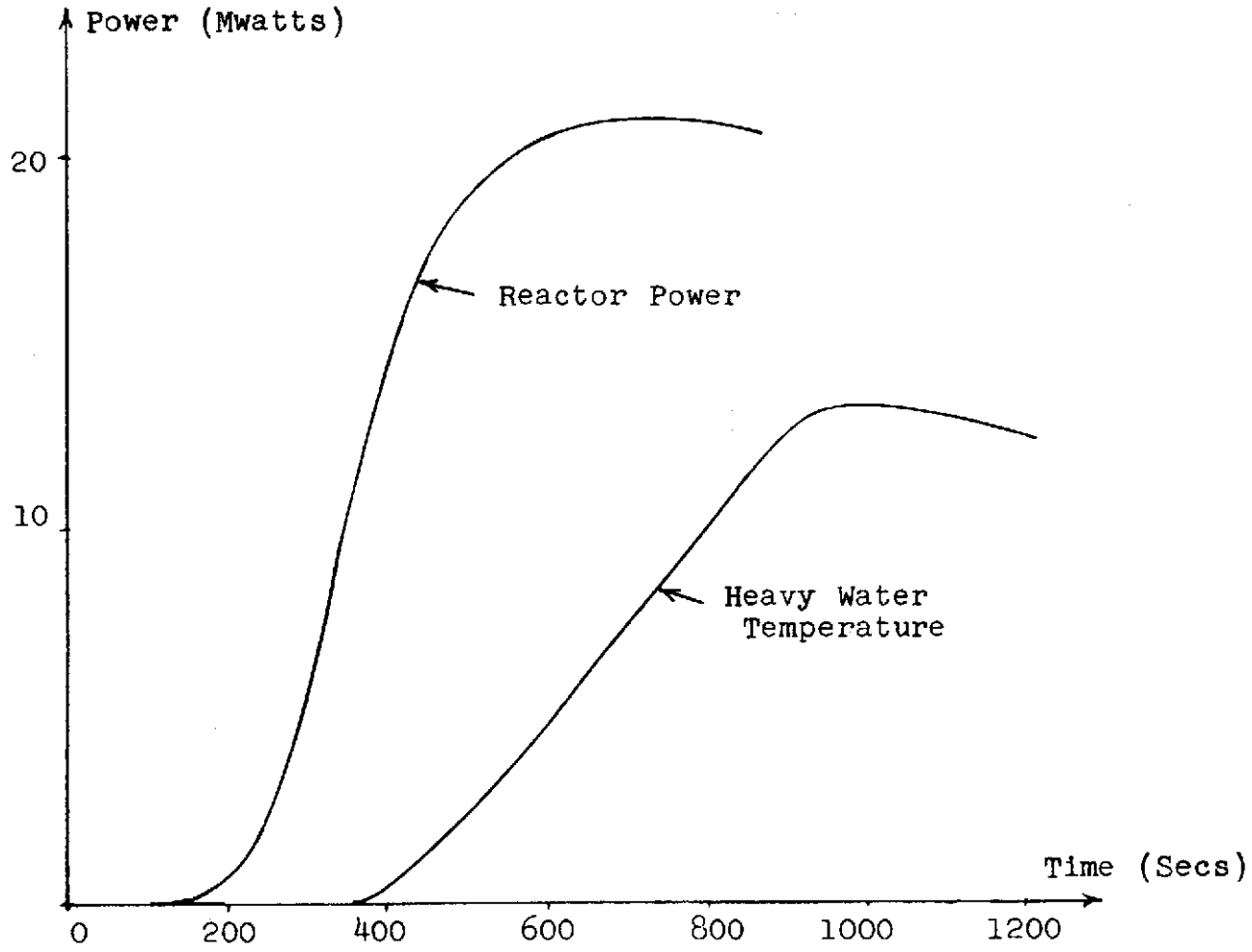


Fig. 1

the increase in the temperatures of fuel and heavy water. However much reactivity was initially added and however fast the initial power rise, the increase in power would slow down somewhat when the temperatures start to increase. This rapid loss in reactivity is sometimes known as PROMPT POISON.

The amount of prompt poison is often measured in terms of the reactor POWER COEFFICIENT. This is defined as the change in reactivity per unit increase in power.

When the reactivity <u>decreases</u> with <u>increase</u> of temperature, the reactor is said to have a NEGATIVE TEMPERATURE COEFFICIENT.

If the reactivity had <u>increased</u> with <u>increase</u> of temperature, then the temperature coefficient would be positive.

<u>The Temperature Coefficient of Reactivity</u> may be defined as the milli-k change in reactivity per $1^{o}F$ increase in temperature.

Temperature changes occur more or less independently, in the fuel, the heat transport system and the moderator and therefore there will be temperature coefficients for each separate effect. It is very desirable that the overall temperature coefficient for a reactor be negative to provide the self-regulating feature illustrated for NRX.

Changes in reactivity with change in temperature occur for a number of reasons which are discussed below.

Factors Causing Reactivity Changes With Temperature

Factors which cause changes in reactivity when the temperature increases may be divided into three main categories as follows:

(a) Mechanical changes which may in turn be due to:

(1) decrease in density of the moderator and heat transport fluid, allowing the neutrons to move further. The neutrons are not slowed down so rapidly nor are they captured so quickly, ie, L and L_s increase and the chances of the neutrons escaping are higher. $L \uparrow, L_s \uparrow$

(2) the fuel elements expanding with temperature, and this reduces the amount of heat transport fluid surrounding the fuel. If the heat transport fluid is light water this will result in a decrease in neutron capture and a consequent increase in reactivity. With heavy water in the heat transport system the change in reactivity will be positive if the reactor is overmoderated, ie, if the moderator to fuel ratio is greater than that required for maximum reactivity. If the reactor is undermoderated there will be a decrease in reactivity. $k \uparrow$

(b) Direct Nuclear Effects - This is the effect commonly known as DOPPLER BROADENING. It is usually stated that resonance capture occurs in U-238 for certain neutron energies but this implies that the target nucleus is at rest. The resonance is actually determined by the relative velocity of the neutrons and the target atoms. If the uranium is hot the atoms are vibrating, and a neutron which would be outside the resonance peak, had the uranium atoms been at rest, may encounter an atom which is moving at the necessary speed to put their relative velocity in the peak. Thus the neutron, which might have escaped in cold fuel, is captured and there is a decrease in the resonance escape probability and in the value of k_∞ due to this so-called Doppler Broadening of the resonance. $p \downarrow$ $k_\infty \downarrow$

(c) Indirect nuclear effects which may be due to:

(1) changes in the so-called NEUTRON TEMPERATURE. The thermal neutrons in a reactor have a distribution of energies approximating to the Maxwellian distribution of energies of molecules, with a characteristic temperature somewhat above the reactor temperature. This effective neutron temperature has been estimated to be equal to the moderator temperature plus ¼ of the temperature difference between moderator and coolant. Thus any increase in temperature of fuel, heat transport fluid or moderator will cause an increase in the neutron temperatures and an increase in the energies of thermal neutrons entering the fuel. In heavy water the main result is to reduce the absorption and thus increase k. In the fuel the ratio of absorption to fission changes causing a decrease in η for uranium and an increase in η for plutonium.

(2) changes due to plutonium buildup - Pu-239 has a resonance at 0.3 ev, so that the absorption in Pu-239 is increased relative to the other components as the fuel is irradiated. This produces a positive change in reactivity. Hence, in a reactor which may have a negative temperature coefficient with fresh fuel, the temperature coefficient will increase and may become zero or positive as the irradiation of the fuel increases, eg, the moderator temperature coefficient in NPD was estimated to be -0.067 mk per °F for fresh fuel and only +0.013 mk per °F for fully irradiated fuel.

It was mentioned earlier that reactivity changes could occur due to temperature changes in fuel, heat transport fluid or moderator and that there would be a temperature coefficient of reactivity associated with each component. These will now be discussed further.

Fuel Temperature Coefficient of Reactivity

The reactivity changes due to increased fuel temperatures are mainly due to two of the effects discussed above, namely:

(a) Increase in the effective temperature of the thermal neutrons in the fuel causing a decrease in η for fresh fuel and an increase in η for fully irradiated fuel.

(b) An increase in resonance capture, with a resulting decrease in p due to Doppler Broadening.

In a reactor using the same type of fuel throughout, the fuel temperature coefficient would be expressed as the milli-k change in reactivity per °C (or °F). Thus, the expected values, due to each of the above effects, for equilibrium fuel in Douglas Point are as follows:

Neutron temperature	+0.0036 mk/°C
Doppler Effect	-0.014 mk/°C
Total fuel temperature coefficient	-0.0104 mk/°C

In a reactor such as NPD, where 7 and 19 element fuel is used, the effects differ for the two types of fuel element. It is, therefore, more convenient to express the fuel temperature coefficient, for the reactor as a whole, as the change in reactivity per 1% change in power. Since changes in power are reflected in changes in the average fuel temperature, T_f, and in the average heat transport temperature T_c, it is convenient to express the coefficient in:

$$\text{mk/\%}\ \Delta(T_f - T_c)$$

ie, as the change in reactivity per 1% change in $(T_f - T_c)$.

The values in NPD for fresh fuel and fuel irradiated to 7200 Mwd/tonne U are as follows:

	Fresh Fuel	Irradiated Fuel	
Neutron temperature	-0.0121	+0.0085	mk/% $\Delta(T_f - T_c)$
Doppler effect	-0.0272	-0.0272	mk/% $\Delta(T_f - T_c)$
Total	-0.0393	-0.0187	mk/% $\Delta(T_f - T_c)$

This table also shows the change that occurs in the neutron temperature effect, and consequently on the total fuel temperature coefficient, because of the plutonium buildup in the irradiated fuel.

When the fuel temperature coefficient is negative excess reactivity must be provided in the reactor to counteract the decrease in reactivity that occurs as the fuel heats up when reactor power is increased. Such a requirement may determine whether or not full power can be achieved under certain circumstances, particularly following a reactor trip. The buildup of xenon, following a reactor trip, limits the time during which the reactor can return to critical, as indicated in a later lesson. If criticality is achieved towards the end of this time limit and the fuel, in the meantime, has cooled down, there is insufficient excess reactivity left to allow for the decrease that occurs as the fuel heats up. Consequently the fuel temperature cannot be allowed to increase and the power cannot be raised. Since the xenon buildup cannot be removed until about 70% of full power is achieved, the xenon continues to build up and the reactor becomes subcritical.

Heat Transport Temperature Coefficient of Reactivity

An increase in the temperature of the heat transport system will result in a change of reactivity because:

(a) the temperature increase again causes an increase in neutron energy with the same results as with the fuel temperature coefficient above.

f ↑
ε ↑
p ↓

(b) a reduction in density causing an increase in f and ϵ and decrease in p, the resonance escape probability.

The overall heat transport temperature coefficient is usually negative with fresh fuel but becomes less negative as plutonium builds up. Thus, in NPD the measured value of this coefficient with fresh fuel was -0.020 mk/°F whereas the present value with a close-to-equilibrium core is -0.003 mk/°F. The expected value with the Douglas Point equilibrium fuel is +0.023 mk/°C or about +0.013 mk/°F.

The negative value for this coefficient compounds the problem of returning to high power following a reactor trip. It is important that, should such a trip occur, the heat transport temperature be kept as high as possible until the reactor is back at high power. This in turn may require isolation of the steam generator from the turbine so that heat from the heat transport system is not being used to produce steam for the turbine.

If the algebraic sum of the fuel and heat transport temperature coefficients is positive, the reactivity decreases as reactor power decreases and the above problem no longer exists. However, such a system is less stable since a transient increase in power leads to a rise in temperature which results in an increase in reactivity which tends to raise the power further.

Moderator Temperature Coefficient of Reactivity

Changes in reactivity with increase in moderator temperature are due to:

(a) increase in the effective neutron temperature with the same results as with the fuel temperature coefficient above.

L^2 ↑ L_s^2 ↑
p ↓ f ↑

(b) decrease in moderator density with resulting increase in L^2 and L_s^2 and hence in neutron leakage. There is also a consequent decrease in p and an increase in f.

With new fuel, the moderator temperature coefficient is negative. As plutonium builds up it becomes less negative and may even become slightly positive because the increase in neutron energy increases fission capture in plutonium, whereas it decreases them in U-235.

The measured value in NPD with fresh fuel was -0.04 mk/°F and the manner in which this value was expected to change with fuel burnup is as shown in Fig. 2.

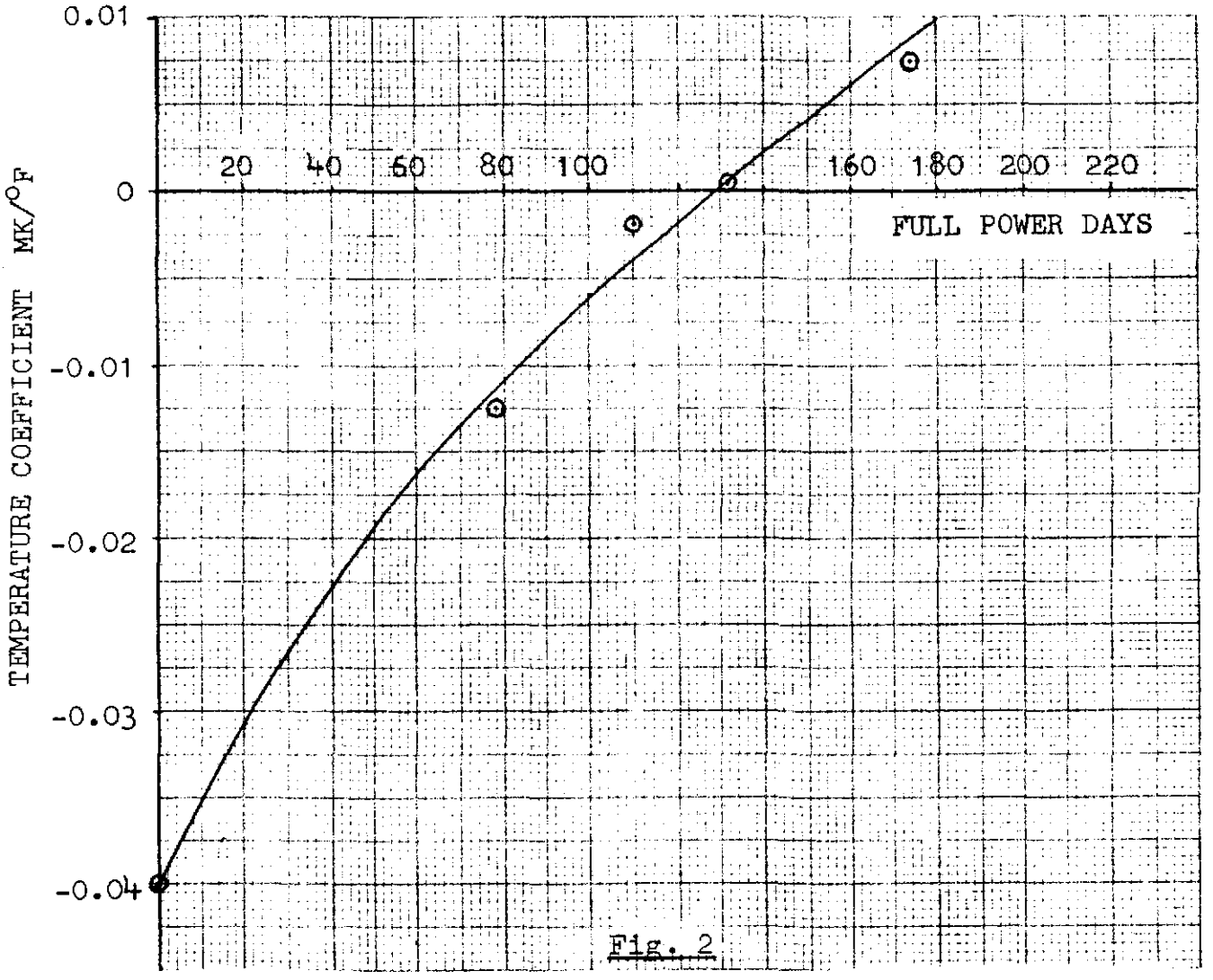


Fig. 2

After 300 to 400 full power days of operation the measured walue was +0.006 mk/°F. There are indications since then that the value has decreased back to zero but, nevertheless, it has become less negative as plutonium has been built up in the fuel.

A negative moderator temperature coefficient would allow some additional reactivity to be obtained, to help counteract a xenon transient following a reactor trip, by cooling the moderator.

Effects Due to Void Formation

Voids may be formed if either the moderator or heat transport system boils. This could be caused by an increase in heat generation, a decrease in cooling flow or a reduction in pressure due to a failure in the system. Generally, if a reactor is overmoderated, ie, with moderator/fuel ratio in excess of that required to just thermalize the neutrons, then a void formed in the moderator or in heavy water heat transport fluid, will cause an increase in reactivity. When the reactor is not overmoderated, then an increase in reactivity can still result. The size of the void and its location are important in deciding whether an increase or decrease in reactivity results from the formation of the void.

The void coefficient of reactivity is defined as the change in reactivity per 1% change in water volume.

Excessive positive or negative void coefficients are to be avoided if possible. An excessively large positive coefficient will cause large power surges, during the void formation, which are likely to cause severe damage to the reactor if the protective system does not respond rapidly enough.

Excessive negative coefficients, on the other hand, cause a rapid decrease in power when the void is formed, which is then corrected for by the regulating system. Then, when the void fills, a power surge again results.

Formation of voids in the heat transport system is of more concern than if they are formed in the moderator system. The heat transport system is pressurized to avoid boiling. However, an increase in fuel channel temperature, due to a power increase or decreased coolant flow can cause boiling. The resulting void could cause a large increase in reactivity, followed by a further increase in power, just when it was not wanted. A break in the heat transport system could cause loss of fluid with the same results due to drop in system pressure.

ASSIGNMENT

1. (a) What is meant by the statement that a reactor has an overall negative temperature coefficient?

   (b) Why is it desirable that a reactor have a negative, rather than a positive temperature coefficient?

2. Define temperature coefficient of reactivity.

3. (a) What factors cause the reactivity to change when the fuel temperature increases and what sort of changes does each factor cause?

   (b) What limitations might a negative fuel temperature coefficient impose on reactor operation?

4. What factors cause changes in reactivity when the heat transport temperature increases?

5. (a) What factors cause changes in reactivity when the moderator temperature increases?

   (b) Why does the moderator temperature coefficient become more positive as the fuel burnup increases?

6. (a) Define the void coefficient of reactivity.

   (b) Why is it undesirable to have excessive positive or negative void coefficients? power surges.

A. Williams

Overmoderated - $\frac{\text{Mod}}{\text{Fuel}}$ ratio $>$ than that needed to just thermalize the neutrons.

Nuclear Theory - Course 127

EFFECTS DUE TO FISSION PRODUCT ACCUMULATION

The temperature effects considered in the previous lesson are relatively short-term effects, ie, the temperature changes, and the resulting reactivity changes occur soon after the change in power occurs. Long-term effects arise due to changes occurring in the fuel as a result of fuel irradiation. These changes are:

(a) Accumulation of fission products in the fuel, and

(b) Burnup of U-235 and build-up of plutonium.

The first of these can also cause a transient change in reactivity on reactor shutdown which can affect the availability of the reactor in a base load station. The gradual and the transient change in reactivity due to (a) will be considered in this lesson.

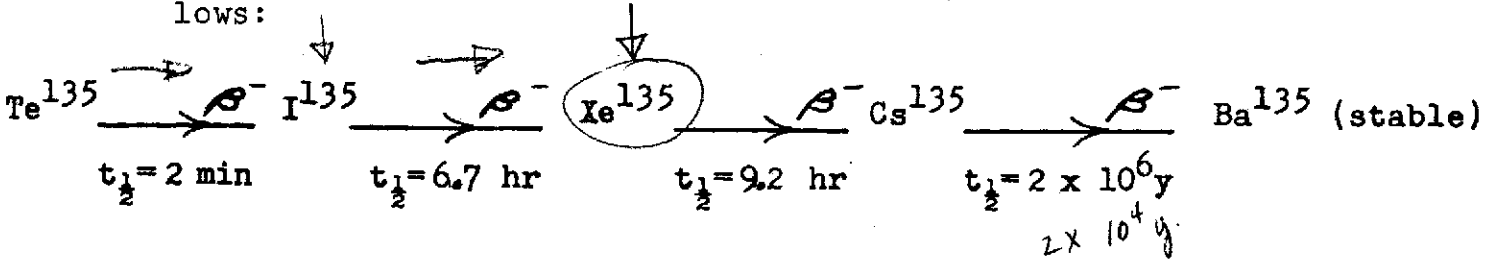
The Build-up of Xenon Poison

All fission products are classified as reactor poisons because they all absorb neutrons to some extent. However, the two most important poisons by far are Xenon-135 and Samarium-149. They have very large thermal neutron capture cross sections and they, therefore, cause substantial changes in reactivity as they build up in the fuel. The one important difference between them is that Xe-135 is radioactive, whereas Sm-149 is stable. This is why they will be discussed separately.

Xenon-135 is produced in the fuel in one of two ways:

(a) Directly as a fission product. About 5% of the total Xe-135 is produced this way.

(b) Indirectly from the decay of Iodine-135, which in turn is produced as a fission product or from the decay of the fission product Tellurium-135. The decay chain is as follows:



Xe-135 is removed from the reactor in two ways:

(a) By decaying to Caesium-135, as shown above, and

(b) By capturing thermal neutrons and forming Xe-136. This Xe-136 is much less of a poison than Xe-135 and its formation can be considered as a complete removal of poison.

When the reactor is first started up, the Xe-135 is formed by the direct method only, since the I-135 has not as yet been formed. So its rate of formation is low. As the iodine concentration grows, the rate of formation of xenon increases and, since its rate of removal is still low, the xenon concentration increases more and more rapidly. However, as the xenon concentration increases its decay rate increases, and its probability of capturing neutrons increases. So, as the xenon builds up, its net rate of removal increases until eventually the rate of removal becomes equal to the rate of formation. Xe-135 is removed just as fast as it is produced and from then on its concentration remains constant. This final constant concentration is known as the equilibrium xenon concentration. The growth of xenon, up to its equilibrium concentration, is shown graphically by the portion AB of Fig. 1.

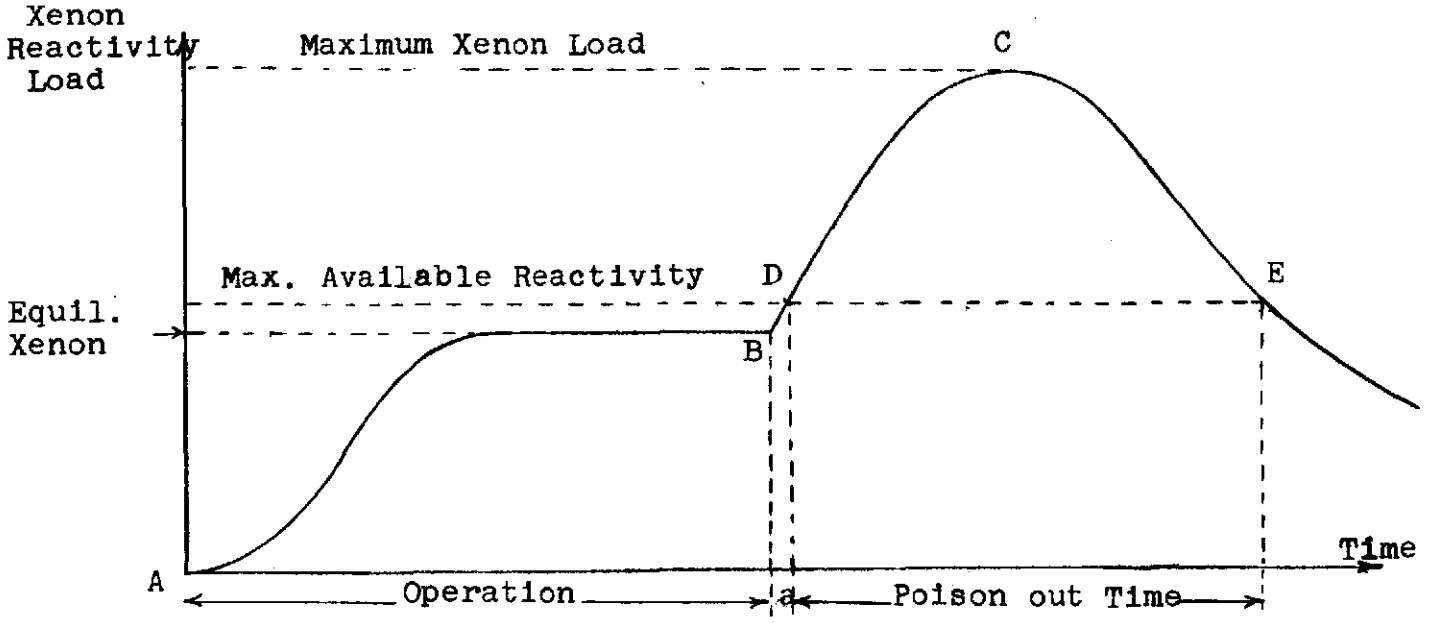


Fig. 1

The slow initial rate of growth of the Xe-135 is very exaggerated in Fig. 1. The actual xenon curve for NPD is shown in Fig. 2. The initial slow rate of growth is due to the fact that the xenon is only being produced directly as a fission product because the I-135 has not as yet been produced.

It may be seen from Fig. 2 that it takes 60 to 70 hours for the xenon concentration to reach its equilibrium value but that it is up to 90% of the equilibrium value for a particular power level in about 25 hours of reactor operation at this power.

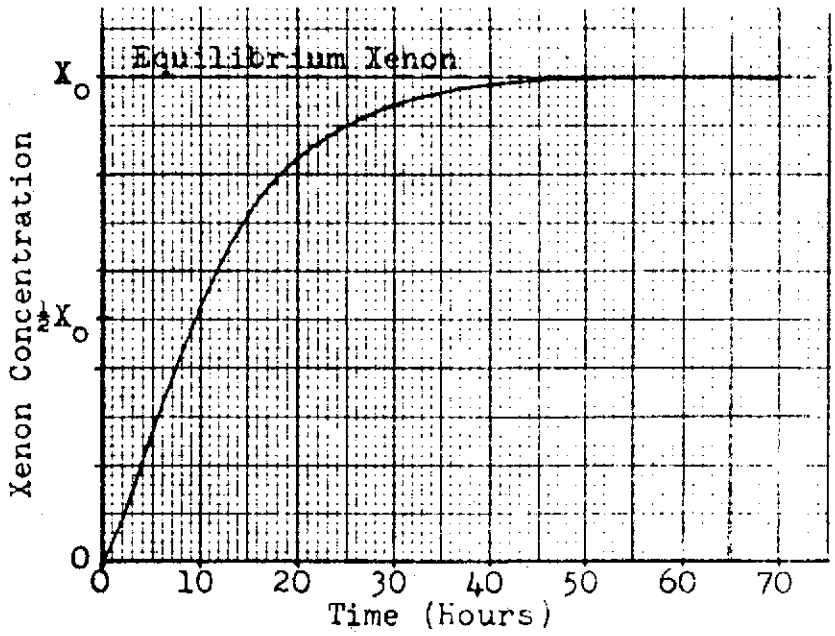


Fig. 2

The POISONING, P, due to the absorption of thermal neutrons by Xe-135, is defined as the ratio of the number of thermal neutrons absorbed by Xe-135 to the number absorbed in fissile material.

The equilibrium poison, P_o, for natural uranium fuel, is given by:

$$P_o = \frac{1.04 \times 10^{-19} \phi}{(2.1 \times 10^{-5}) + (3 \times 10^{-18} \phi)}$$

where ϕ is the thermal neutron flux.

Therefore, for a particular reactor, the poisoning, P_o, depends only on the operating flux, ϕ. The variation of P_o with ϕ in a natural uranium reactor is shown in Fig. 3. The graph shows that, if the flux is less than $10^{12} n/cm^2/sec$, the poisoning is small. There is, then, a rapid increase in P_o up to $\phi = 10^{14} n/cm^2/sec$, after which the rate of increase slows down. P_o eventually reaches a limiting value of about 0.035.

Because Xe-135 absorbs neutrons, which could otherwise be used for fission, there is a reduction in reactivity due to the growth of xenon. This reduction in reactivity is known as the XENON REACTIVITY LOAD. Enough reactivity must be built into the reactor in order that the regulating system can compensate for the loss of reactivity as the xenon concentration increases. The reactor must, therefore, be bigger than the critical size required with fresh fuel. However, when the reactor is first started up, the xenon concentration is either zero or very small.

Thus, if moderator level reactivity control only is used, as at NPD, the moderator level would be well below its normal operating level. This results in a reduction in the permissible power and a loss of energy revenue. In Douglas Point and Pickering, boric acid is used as a neutron absorber or poison in the moderator to enable the reactor to operate at full tank and, therefore, at full power even when the xenon load is low. As the xenon grows, the boric acid is removed with ion exchange columns.

The growth of xenon load to equilibrium is, of course, identical to that of the xenon concentration and it is the growth of the xenon load that is shown in Fig. 1.

The xenon load varies with the flux in the same way as the poisoning, shown in Fig. 3.

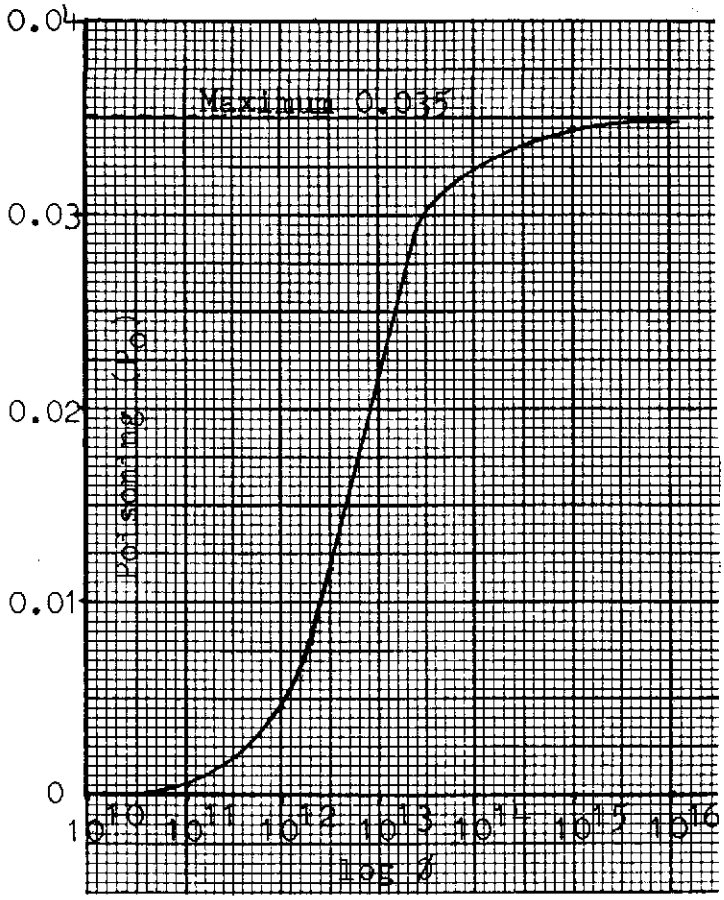


Fig. 3

Xenon Build-up During Reactor Shutdown

An important aspect of poisoning arises on reactor shutdown. Suppose the reactor trips, or is shut down, at the point B in Fig. 1. The neutron flux decreases to a low value and the direct production of I-135 and Xe-135 as fission products cease. However, the I-135 already present in the fuel continues to decay to Xe-135 and so the formation of xenon continues at a rate a little lower than at reactor power.

Removal of Xe-135 by neutron absorption stops, whereas its removal by decay continues at the same rate as before. The net result is that the rate of production is now greater than the rate of removal and the Xe-135 concentration and reactivity load starts to increase along the portion BC of the curve in Fig. 1.

However, as the Xe-135 concentration increases, its rate of decay increases. Also its rate of formation decreases because

of the decrease in the amount of I-135 remaining. Eventually, the rate of removal becomes equal to the rate of formation and then becomes greater than the rate of formation. The net result is that, as shown in Fig. 1, the xenon load reaches a maximum value at C, and then starts to decrease, eventually decaying exponentially with the Xe-135 half-life. Such a build-up of xenon is known as the Xenon Transient. The maximum xenon concentration occurs about 9 to 11 hours after the trip. The maximum xenon load may well be double the equilibrium load and could reach a much higher value.

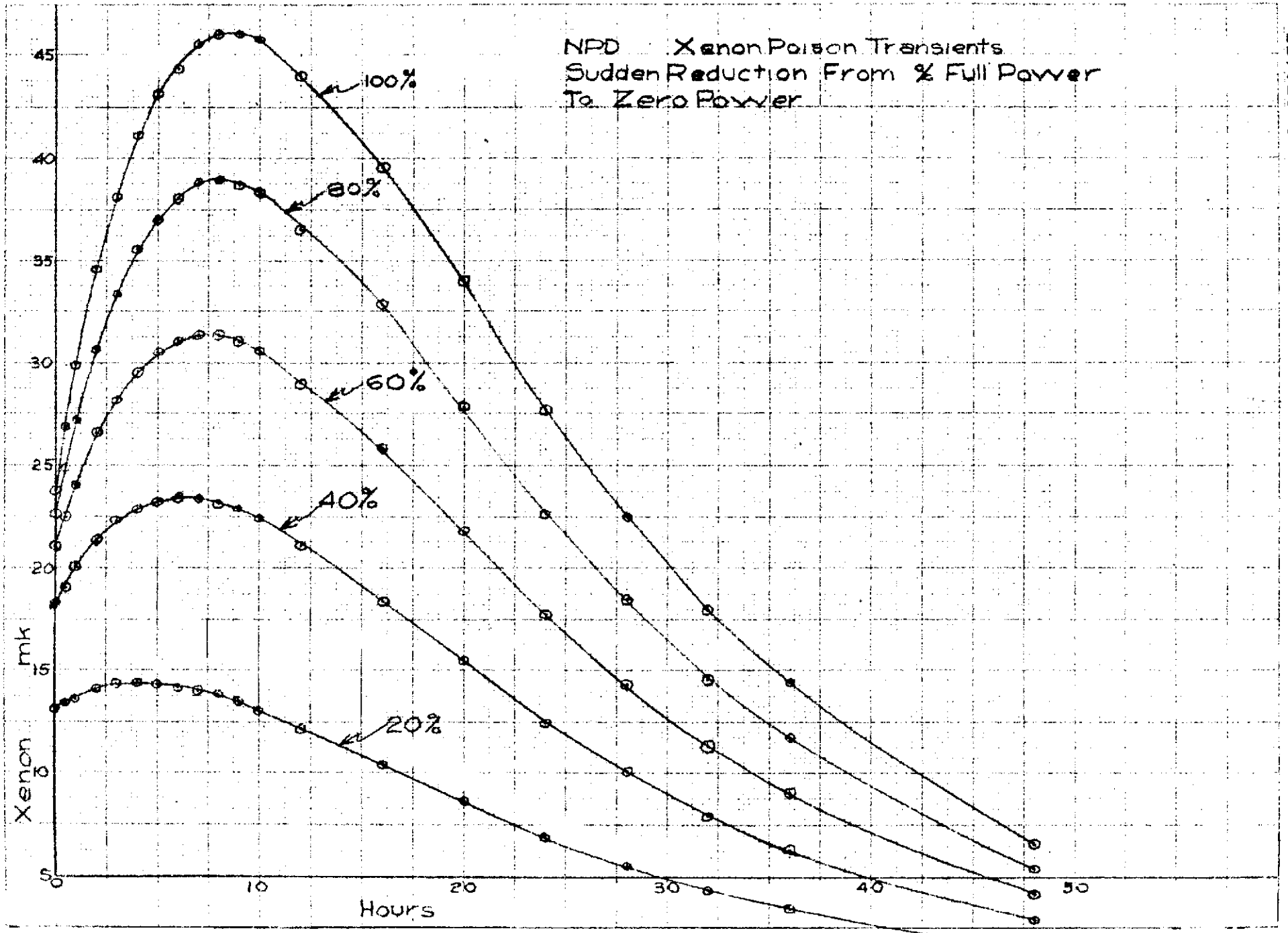
The maximum xenon reactivity load depends on the thermal neutron flux, or the power, at the time of the trip. Fig. 4 shows how the xenon transient varies when a reactor is shut down from different percentages of full power. It should also be noted that the transient continues to increase beyond flux values of $10^{14} n/cm^2/sec$ and does not level out in the manner of the equilibrium load.

The initial rate of rise of the xenon load, after a reactor shutdown, is also proportional to average flux or power at the time of the shutdown.

If the reactivity built into the core is only just enough to overcome the equilibrium xenon load, then it may well be 24 to 30 hours before the xenon decays sufficiently to enable the reactor to be started up again. It is not feasible to provide enough excess reactivity to overcome the peak xenon but sufficient reactivity may be provided to make it possible to start up within half or three-quarters of an hour of a trip, ie, from B to D in Fig. 1. The time period "a", during which enough reactivity is available to overcome the xenon load and start up the reactor, is known as the POISON OVERRIDE TIME or the TIME TO POISON. The time from D to E, during which the reactor cannot be started up because there is not enough reactivity to overcome the xenon load, is known as the POISON-OUT TIME.

In a reactor with a negative moderator temperature coefficient, some of the required additional reactivity can be obtained by cooling the moderator. However, the best method of providing this reactivity is to insert extra fuel, in the form of a booster rod, into the reactor. This avoids increasing the fuel inventory in the core since the booster rod is only used when required.

The poison override time can be extended by a schedule of power reductions, provided that prior warning is given of the intended shutdown. Some loss of energy production is sustained but poisoning out, with its associated greater production loss, is avoided.


NPD Xenon Poison Transients
Sudden Reduction From % Full Power
To Zero Power
100%
80%
60%
40%
20%
Xenon mk
Hours
45
40
35
30
25
20
15
10
5
0
10
20
30
40
50

F1 4

The Build-up of Samarium Poison

Of the stable poisons, the most notable is Samarium-149, which is formed by the decay of Promethium-149, which in turn is formed by the decay of Neodymium-149:

$$Nd^{149} \xrightarrow{\beta} Pm^{149} \xrightarrow{\beta} Sm^{149}\ \text{(stable)}$$

$t_{\frac{1}{2}} = 1.8$ hr for Nd^{149}; $t_{\frac{1}{2}} = 53$ hr for Pm^{149}

∴ no removal from decay

Sm-149 poison reaches its equilibrium in much the same way as Xe-135, as shown in the portion AB of the graph in Fig. 5. Sm-149, being stable, is not removed by decay and its rate of removal by neutron capture is slower than that of xenon. However, its rate of formation is also smaller, with the result that it reaches equilibrium more slowly than xenon. The Sm-149 equilibrium load is only slightly greater than a fifth of that of Xe-135.

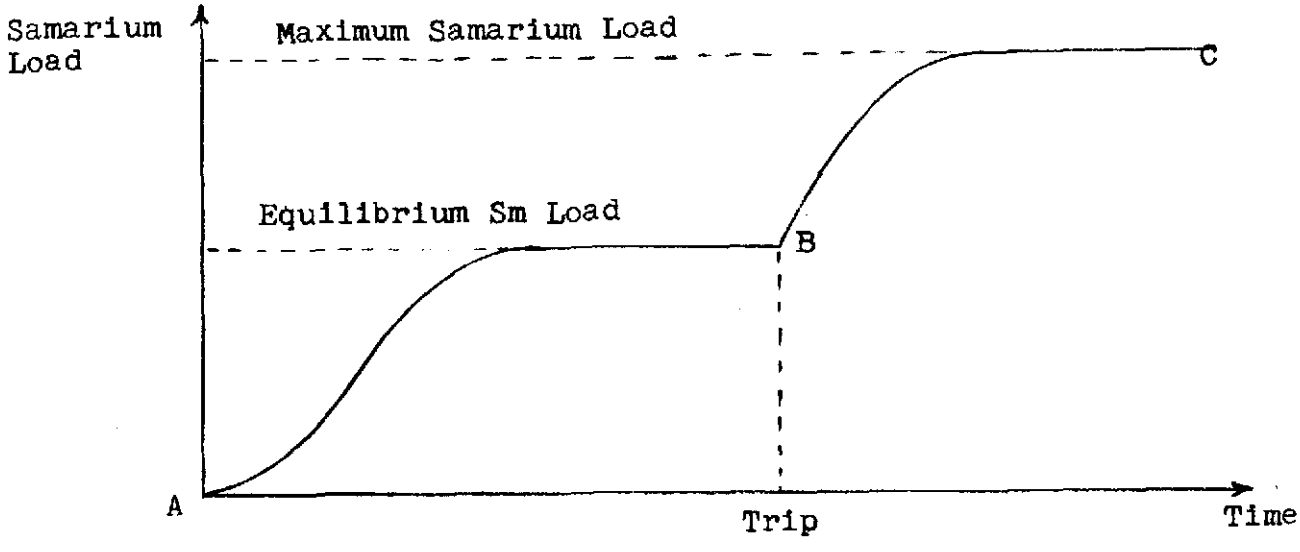


Fig. 5

When the reactor trips, or is shut down, the samarium increases much in the same way as the xenon. However, since there is no Sm-149 decay, it continues to increase to a new equilibrium level, until all the Pm-149 has decayed. The growth of Sm-149, after a trip, is shown in the portion BC of Fig. 5. The maximum Sm-149 load will be two or three times the equilibrium operating load.

The value of the equilibrium samarium load is independent of the operating flux but the maximum transient load does depend on the flux or power at the time of the shutdown.

Note that the only way to reduce the Sm-149 is to start up the reactor and remove Sm-149 by neutron capture. An allowance in the design of the reactor core must be made for the equilibrium Sm-149 load. However, the transient following shutdown is normally no problem since it is balanced by the Pu-239 transient. It was previously shown that, as a result of neutron capture in U-238, Pu-239 is formed by the following decay chain.

$$U^{239} \xrightarrow[t_{\frac{1}{2}} = 0.4 \text{ hr}]{\beta^-} Np^{239} \xrightarrow[t_{\frac{1}{2}} = 56 \text{ hr}]{\beta^-} Pu^{239}$$

The decay of Np-239 takes place at approximately the same rate as decay of Pm-149 and it happens that the enrichment effect of the extra Pu-239 formed after shutdown almost balances the load due to Sm-149.

Following startup there is a slight overall enrichment effect, since the Sm-149 returns to equilibrium more quickly than the Pu-239. This effect is normally masked out by xenon build-up and is not obvious.

ASSIGNMENT

1. (a) Explain briefly why xenon concentration increases to an equilibrium value during reactor operation.

   (b) What is meant by "Equilibrium Xenon Reactivity Load"?

   (c) On what does this xenon load depend?

2. Explain why there is a transient increase in xenon concentration after reactor shutdown and how the increase depends on the operating power prior to shutdown.

3. (a) Explain the terms "Poison Override Time" and "Poison-Out Time".

   (b) What is the significance of using a booster rod in a reactor?

4. (a) Why does samarium build-up, following a shutdown, differ from that of xenon?

   (b) How can the samarium load, in a reactor, be reduced?

   (c) Why is it necessary to make allowance in the design of a reactor for the equilibrium samarium load but not for the transient samarium load?

A. Williams

Poisons	σ_a (barns)	$t_{1/2}$
Xe^{135}	2,700,000	
Sm^{149}	65,000	
Sm^{151}	14,800	
Cd^{113}	19,740	
Eu^{155}	13,600	
Gd^{155}	255,000	

Nuclear Theory - Course 127

EFFECTS OF FUEL BURNUP

The effect of fuel burnup was considered, to some extent, in a previous lesson. During fuel burnup, U-235 is used up and plutonium is produced and later burnt. Consideration was given to the effect of this on the value of k. This will now be discussed in greater detail and other effects also considered.

Conversion and Breeding

Before the effects of fuel burnup can be discussed, it is necessary to consider the production of new fissile material that occurs during the fuel burnup. The only naturally occurring fissile or fissionable material is U-235. However, Pu-239 can be produced from U-238 and U-233 can be made from Thorium-232. U-238 and Th-232 are known as FERTILE material. Pu-241 is another fissile material that is produced by neutron capture in Pu-240, which is in turn produced by neutron capture in Pu-239.

A reactor, in which the fissile material produced from the fertile material is the same as the fissile material being consumed, is known as a BREEDER reactor. Thus, if U-233 was being used as a fuel, in a reactor which also contained Th-232, then U-233 would also be produced from the Thorium.

A reactor, in which the fissile material produced from the fertile material is not the same as the fissile material being consumed, is known as a CONVERTER reactor. A reactor using natural uranium fuel is a converter. It burns U-235 and produces plutonium. This is the type of reactor which is of interest.

The CONVERSION FACTOR, c, is defined as the number of fissile atoms produced for each fissile atom consumed.

If $c = 1$, then for each U-235 atom fissioned, one Pu-239 atom will be produced. Under these conditions every fissile atom burnt is replaced and there is no depletion of fissile atoms in the fuel. Thus, provided that there were no other physical limitations, every fissile and fertile atom in the fuel could be used, eg, all the U-235 atoms in natural uranium would be used and all the U-238 atoms would be converted to plutonium and the plutonium burnt. The conversion factor is not, however, as high as this in a power reactor. It is more likely to be around 0.75 or 0.8.

Alternative definitions of breeders and converters are based on the value of c.

A converter is a system in which $c < 1$.

A breeder is a system in which $c \geq 1$.

Now η is the number of neutrons produced per neutron absorbed in the fuel and 1.0 of these neutrons must be available to cause further fissions and maintain the chain reaction. So, the maximum possible number of neutrons available for breeding or conversion is $\eta - 1$ and this disregards neutron losses by leakage and absorption in reactor material. If w is the number of neutrons lost from the system, per fission, by leakage or absorption,

$$c = \eta - w - 1$$

For good conversion, or breeding, η must be as large as possible and w kept as small as possible. The following table lists the value of η for fast and thermal neutrons for the three fissile materials

	U-235	U-233	Pu-239
η for fast neutrons	2.46	2.54	2.88
η for thermal neutrons	2.08	2.31	2.03

It may be seen that, in thermal reactors using U-235 or Pu-239 as fuel, $\eta - 1$ is only just greater than 1.0 and, therefore, net breeding or conversion cannot be achieved. It is, however, theoretically possible in a thermal reactor using U-233 fuel and Th-232 as the fertile material.

In a fast reactor, on the other hand, breeding or conversion is possible with all three fissile materials and is particularly attractive with Pu-239.

Effect of Conversion on Fuel Burnup

Fuel burnup may be defined in one of three ways:

(a) Burnup is the percentage of the original fissile atoms burnt.

(b) Burnup is the percentage of the total fuel atoms burnt.

(c) Burnup is the heat extracted (in Megawatt days) per tonne (10^6 gm) of fuel.

Thus, for a fuel of enrichment E, (ie, having E atoms of fissile material and (1 - E) atoms of fertile material), b% fissile atom burnup = Eb% fuel burnup = 10,000 Eb Mwd/tonne fuel.

For natural uranium fuel, E = 0.00715 and so:

b% fissile burnup = 0.00715 b% fuel burnup = 71.5 b Mwd/tonne

If all the fissile atoms in natural uranium were burnt (b = 100), the burnup would be 0.715% fuel atoms or 7150 Mwd/tonne of fuel. However, conversion and breeding produce other new fissile atoms in the fuel which can also be burnt.

If c is the conversion factor, the maximum or ULTIMATE burnup that can be achieved is $\frac{100}{1 - c}$ % of original fissile atoms.

So with c = 0.8

Ultimate burnup = $\frac{100}{1 - 0.8}$ = 500% fissile atoms

= 3.57% of all fuel atoms

= 35,700 Mwd/tonne

No fissile material would then be left but roughly (100 - 3.57)% or 96.43% of U-238 would remain. This maximum burnup cannot be achieved in practice because:

(a) Too much excess reactivity would have to be built into the reactor to allow for the fuel depletion, ie, the reactor would have to be too big.

(b) The fuel integrity would be questionable, ie, fuel failure would be very likely to occur due to buildup of pressure of fission product gases and fuel distortion, unless an excessively thick fuel sheath was used.

Using UO_2 fuel and on-power refuelling, a burnup of 10,000 Mwd/tonne of uranium is a reality and serious consideration is being given to burnups as high as 15,000 Mwd/tonne of uranium or 1.5% of all uranium atoms in the fuel or 210% of U-235 atoms.

Effect of Burnup on Reactivity

There are two aspects of the effect of burnup on reactivity and these are:

(a) Change in Total Reactivity Due to Burnup

The curve in Fig. 1 shows the change in reactivity with burnup allowing for the accumulation of poisons.

This combines the curve of k vs burnup, in lesson 127.10-6 with the reduction in reactivity due to poison buildup. The curve shows:

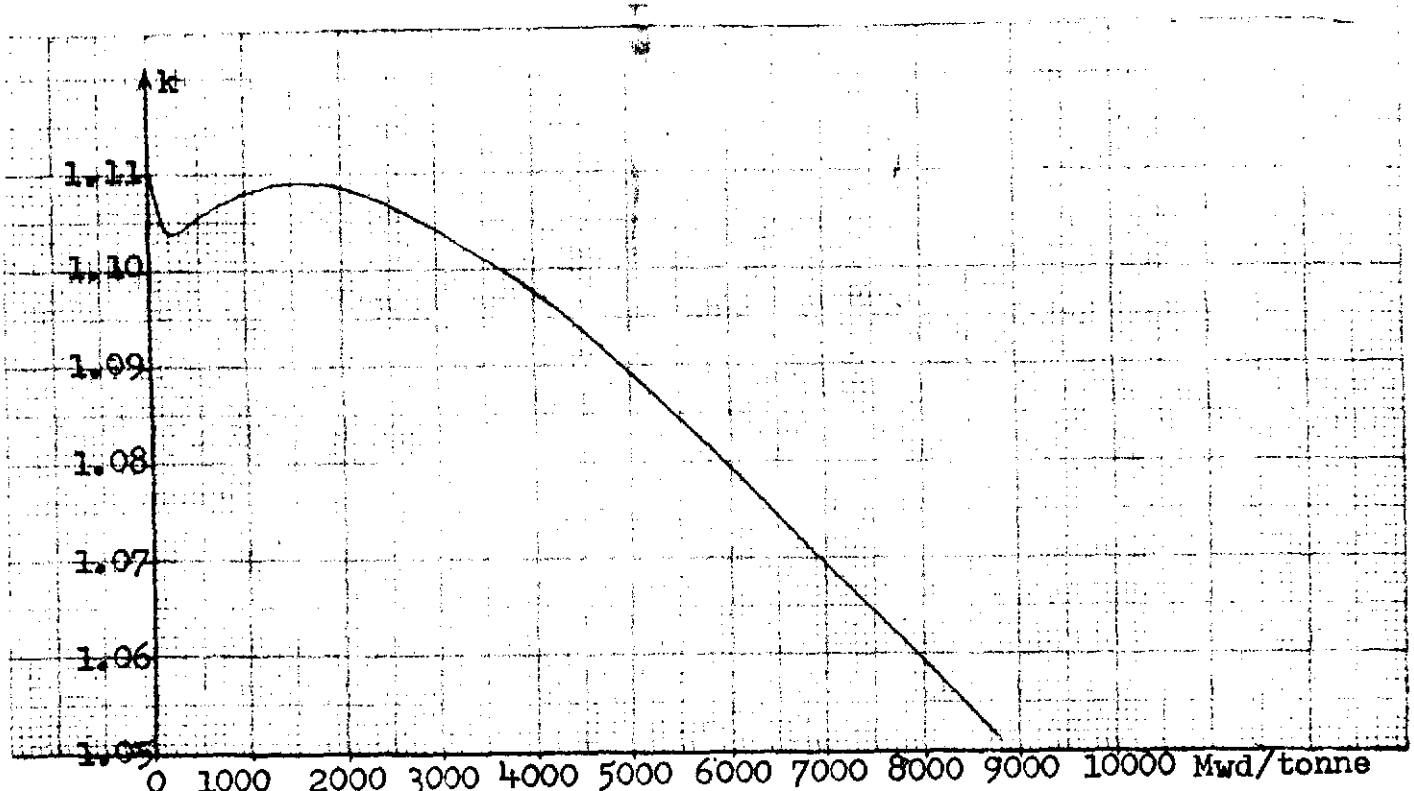


Fig. 1

(1) A small decrease in δk initially due to the fact that the buildup of poisons masks out the effect of plutonium buildup.

(2) With a conversion factor of about 0.8 or less, the U-235 is being used faster than the plutonium is being produced. Even so, because of the higher fission cross section of Pu-239 the effect of U-235 burnup is more than compensated for. An increase in reactivity occurs which is more than enough to compensate for the decrease in reactivity due to poison buildup anda net increase in reactivity results up to a burnup of 1500 Mwd/tonne.

After 1500 Mwd/tonne the burnup of plutonium itself and the production of non-fissionable Pu-240 (from neutron capture in PU-239) causes the reactivity to decrease.

(3) A burnup of 10,000 Mwd/tonne involves a net loss of 70 mk, which must, therefore, be available in the core if this burnup is to be attained.

(b) Modification in the Reactivity Temperature Coefficients

This aspect of burnup has also been mentioned. One effect of a temperature increase in the fuel, heat transport system or the moderator is due to the fact that the thermal neutrons enter the fuel with a higher speed or increased energy. With fresh fuel, the predominant fissile material is U-235 and the fission cross section for U-235 decreases as the neutron energy increases above 0.025 ev. This means that radiative captures increase relative to the fission captures in the fuel and, consequently, the value of η decreases in the four factor formula, ie, fewer neutrons are produced for each neutron absorbed in the fuel. The net result, due to this effect alone, is a decrease in reactivity due to an increase in temperature.

If this decrease in reactivity is not counterbalanced by an increase in reactivity, due to other effects, then the reactor has a negative temperature coefficient.

As the fuel is burnt up, the U-235 is depleted but the plutonium content increases. With plutonium, the fission cross section increases for neutron energies above 0.025 ev. This is due to the fact that the plutonium fission cross section has large values at some resonance energies above 0.1 ev. So the higher the neutron energy becomes, the more neutrons there will be having energies at or near to these fission resonances. Therefore, with plutonium, the values of η and the reactivity increase as the neutron energy increases, due to an increase in temperature. Thus, as the U-235 becomes depleted and the plutonium concentration increases, the temperature coefficient of reactivity becomes less negative and more positive.

This type of change is especially noticeable when the moderator temperature is changed. With fresh fuel, the moderator temperature coefficient is negative. An <u>increase</u> in reactivity can be obtained, when it may be required, say to prevent a poison out, by <u>cooling</u> the moderator. However, as burnup increases, cooling the moderator may cause very little reactivity change or even a <u>decrease</u> in reactivity.

Effect of Burnup on Reactor Control

Reactor control is possible only because the delayed neutrons, despite being such a small fraction of the neutron population, cause a substantial increase in the average lifetime of all neutrons. The delayed neutron yields from U-235 are such that the average lifetime increases from 0.001 sec (for prompt neutrons) to 0.1 sec. How do the yields from Pu-239 compare with those from U-235?

The following table shows the comparison and also the product of yield x average life in both cases.

$t_{\frac{1}{2}}$ (sec)	Av.life (sec)	U-235		Pu-239	
		% yield	Yield x Av.life	% yield	Yield x Av.life
55.6	80.20	0.025	2.00	0.014	1.12
22.0	31.70	0.166	5.26	0.105	3.33
4.51	6.51	0.213	1.39	0.126	0.82
1.52	2.19	0.241	0.53	0.119	0.26
0.43	0.62	0.085	0.05		0.00
0.05	0.07	0.025	0.00		0.00
Prompt	0	99.245	0.00	99.636	0.00
Total		100.000	9.23	100.000	5.53

ℓ_1 = Average lifetime with U-235 = $\frac{9.23}{100} + 0.001 = 0.0924$ sec

ℓ_2 = Average lifetime with Pu-239 = $\frac{5.53}{100} + 0.001 = 0.0554$ sec

So for a 1 mk increase in reactivity:

Reactor Period with U-235 = $\frac{0.0924}{0.001} = 92.4$ sec and, in 1 sec, $P = 1.011\ P_o$

Reactor Period with Pu-239 = $\frac{0.0554}{0.001} = 55.4$ sec and, in 1 sec, $P = 1.02\ P_o$

So, for a 1 mk increase in reactivity, the power increases by 1.1% in the first second, with U-235 only, and it increases by 2% in the first second, with Pu-239.

Therefore, reactor control is still feasible with Pu-239 fuel but the response of the control system must be faster. Therefore, in the design of the control system an allowance must be made for the decrease in reactor period as the plutonium concentration increases.

It should be noted that the reactor is prompt critical when $\delta k = 7.55$ mk, with U-235 fuel only. However, when all the U-235 is burnt and the reactor is operating on plutonium fuel only, the reactor is prompt critical when $\delta k = 3.64$ mk. The control system must, therefore, be designed to prevent reactivity values even approaching $\delta k = 3.64$ mk. It is no longer sufficient to keep δk below 7.55 mk.

ASSIGNMENT

1. (a) Explain the difference between a fissile and a fertile material.

   (b) Explain the difference between a breeder and a converter reactor.

2. (a) What is the ultimate fuel burnup that could be obtained with a conversion factor of 0.8?

   (b) Why is it not possible to achieve this ultimate burnup in practice?

3. (a) Why does the reactivity decrease sharply for a short while after operation of the reactor has started?

   (b) Why does the reactivity then start to increase even though U-235 is being used up faster than plutonium is being produced?

   (c) Why does this reactivity increase not continue beyond about 1500 Mwd/tonne of uranium?

4. Explain why, with fresh fuel, poison override time can sometimes be extended by cooling the moderator, whereas fuel burnup prevents this later.

5. (a) Why must a reactor control system response be faster than is necessary with fresh fuel?

   (b) What other limitation must the control system impose to allow for fuel burnup?

A. Williams

Nuclear Theory - Course 127

REACTOR CONTROL

As has been seen from the preceding lessons, there are many factors which change the neutron multiplication factor in a reactor. Changes in temperature cause short-term changes in reactivity, fission product poison loads have to be allowed for, and extra reactivity must be built into the reactor to allow for depletion or "burnup" of fissionable material. Hence, a control system must be provided which must meet the following requirements:

(a) It must keep $k = 1$ and $\delta k = 0$ during steady power operation and, therefore, it must compensate for changes in δk that occur for various reasons.

(b) It must allow δk to become positive or negative for increases and decreases, respectively, in power.

(c) It must decrease k sufficiently to give δk a large negative value for rapid shutdown of the reactor when this is required.

Regulation and Protection

Because of the general requirements outlined above, the control system must perform two functions which are:

(a) Regulation which involves small changes in reactivity to maintain the power at some predetermined level or to change the power as required. The regulating system is, therefore, the means by which the reactor is started up, operated at some desired power level and shut down.

(b) Protection which is the provision of automatic rapid shutdown of the reactor under any circumstances which might prove hazardous to personnel or equipment. Such rapid shutdown, which is known as tripping the reactor, is achieved by means of the protective system. The conditions which would require a reactor trip are:

(1) Uncontrolled and excessive power increase or excursions.

(2) Excessive rate of change of power.

(3) Unsafe faults in the regulating system.

(4) Failure of some process, such as heat transport system flow, process air, etc, which is critical as far as safe reactor operation is concerned.

It should be remembered that the regulating system is, in fact, the first line of protection since it limits reactor power to safe levels. Most trips, by the protective system, are likely to be caused by misoperation of the regulating system.

Methods of Control

All methods of control use some method or other of changing neutron losses in order to change the value of k. There are three general methods of changing neutron losses leading to four methods of control. These three general methods are:

(a) Changing neutron absorption in materials other than the fissile material U-235 or plutonium. This changes the number of neutrons available for fission and changes the value of k.

Changes in neutron absorption are usually achieved by inserting neutron absorbers into the reactor core or withdrawing such absorbers out of the core. The conventional method of control inserts or withdraws control rods made of boron or cadmium, which have large neutron absorption cross sections. When the control rods move further into the core, k decreases. When the rods are withdrawn, k increases.

Rapid reduction of reactivity is achieved by quick and complete insertion of additional safety or shutdown rods, also made of boron or cadmium.

(b) Addition or removal of fuel so that neutron absorption in U-235 nuclei increases or decreases, with consequent increase or decrease in the value of k.

A practical example of reactivity control by this method is the on-power refuelling of Canadian nuclear power reactors. When reactivity decreases because of U-235 burnup, spent fuel is replaced by new fuel to replenish the U-235 and increase k. Also a fuel rod, known as a booster rod, is inserted in the reactor when additional reactivity is required to avoid a shutdown due to poisons.

This addition or removal of fuel can only be used for regulation unless special provisions are made to drop part of the core as a protective measure.

(c) Changing neutron leakage from the reactor, which again increases or decreases the number of neutrons available for fission and, thereby, increases or decreases k. Neutron leakage may be changed in one of two ways:

(1) By increasing or decreasing core size. This may be done by addition or removal of fuel but this is not a very versatile method for continuous control. With a liquid moderator, by far the easiest method is by raising or lowering moderator level to cover more or less fuel. This is a form of MODERATOR LEVEL CONTROL.

(2) By increasing or decreasing reflector thickness. When reflector thickness increases more neutrons are reflected back into the core and more are, therefore, available for fission. A decrease in reflector thickness, on the other hand, reduces k by reducing the neutrons available for fission. Again, the simplest way of achieving this type of variation is by moderator level control. By moving the moderator level up or down, the thickness of moderator above the core which acts as a reflector is increased.

Both the above methods enable rapid reduction of reactivity to be achieved by simply providing the means of emptying the moderator out of the reactor vessel in a few seconds. This is known as "dumping" the moderator, and causes rapid loss in reactivity for protection.

Two or more such control methods can be, and are, used on some reactors.

Advantages and Disadvantages
of Moderator Level Control

Moderator level control offers the following advantages over other methods, such as control rods:

(a) A simple arrangement of valves and helium blowers is used with moderator level control, whereas relatively more expensive reliable drives are required with control rods. Complex circuits interlocking rod withdrawal are necessary and it is necessary for the drives to operate reliably in relatively higher radiation fields.

(b) The use of valves for regulation and in the protective system allows for their operation from a triplicated control system while preserving virtually complete independence of the three control channels. A single failure affects one channel whereas a shutdown only occurs on simultaneous failures in two channels out of three.

The major disadvantages of moderator level control is that resulting from the use of moderator level as a reactivity "shim", as in NPD G.S., to counteract the absence of the xenon poison after a prolonged shutdown. Until the xenon poison builds up to

its equilibrium level, the moderator level is well below its normal operating level. This has the advantage of conserving neutrons by not absorbing them in control rods, but it has the following disadvantages:

(a) Because of the change in thermal neutron flux distribution, resulting from the low moderator level, there is a danger of overheating in some channel, if the reactor is still operated at full power. For this reason there is a reduction in the maximum permissible power. In a large reactor this would result in loss of revenue. Hence, in Douglas Point, boric acid is used as a neutron absorber in the moderator to enable the reactor to operate at full tank and at full power even when the xenon load is low. As the xenon poison grows, the boric acid is removed with ion exchange columns.

(b) The low moderator level results in some fuel channels not being immersed. Spray cooling is then required on the calandria tubes for these channels to prevent stresses due to differential expansion.

It has been said above that rapid reduction of reactivity can be achieved by dumping the moderator. The effectiveness of such a dump in reducing reactivity is shown in Fig. 1. This curve shows the measured reactivity reduction, in the NPD reactor, during the first second following a reactor trip.

The initial 0.25-second delay is due to the time required for the protective system to respond to the trip signal and initiate the opening of the dump valves. Despite this initial delay, the reactivity reduction is very rapid, resulting in a reactivity decrease of about 70 mk in 5 seconds. Such a method of reactivity reduction is therefore very satisfactory in a small reactor such as NPD.

However, there is some doubt as to whether moderator dump should be used for rapid reactivity reduction in large reactors such as those required for the 500-Mwe units in Pickering or, for that matter, in the Douglas Point reactor. Such a dump requires initial rapid movement of several tons of water from the reactor vessel into a dump tank to produce a rapid reactivity decrease during the first second following a trip. This introduces engineering problems in the design of dump ports which will allow such a rapid dump and still support a calandria full of water. It also takes some time to transfer the water back into the reactor vessel and this makes it difficult to return to full power after a trip before the reactor poisons out, unless excessive pumping power is used. Pump-up time, in Pickering, to full calandria is estimated as 50 minutes, whereas the poison override time is 45 minutes.

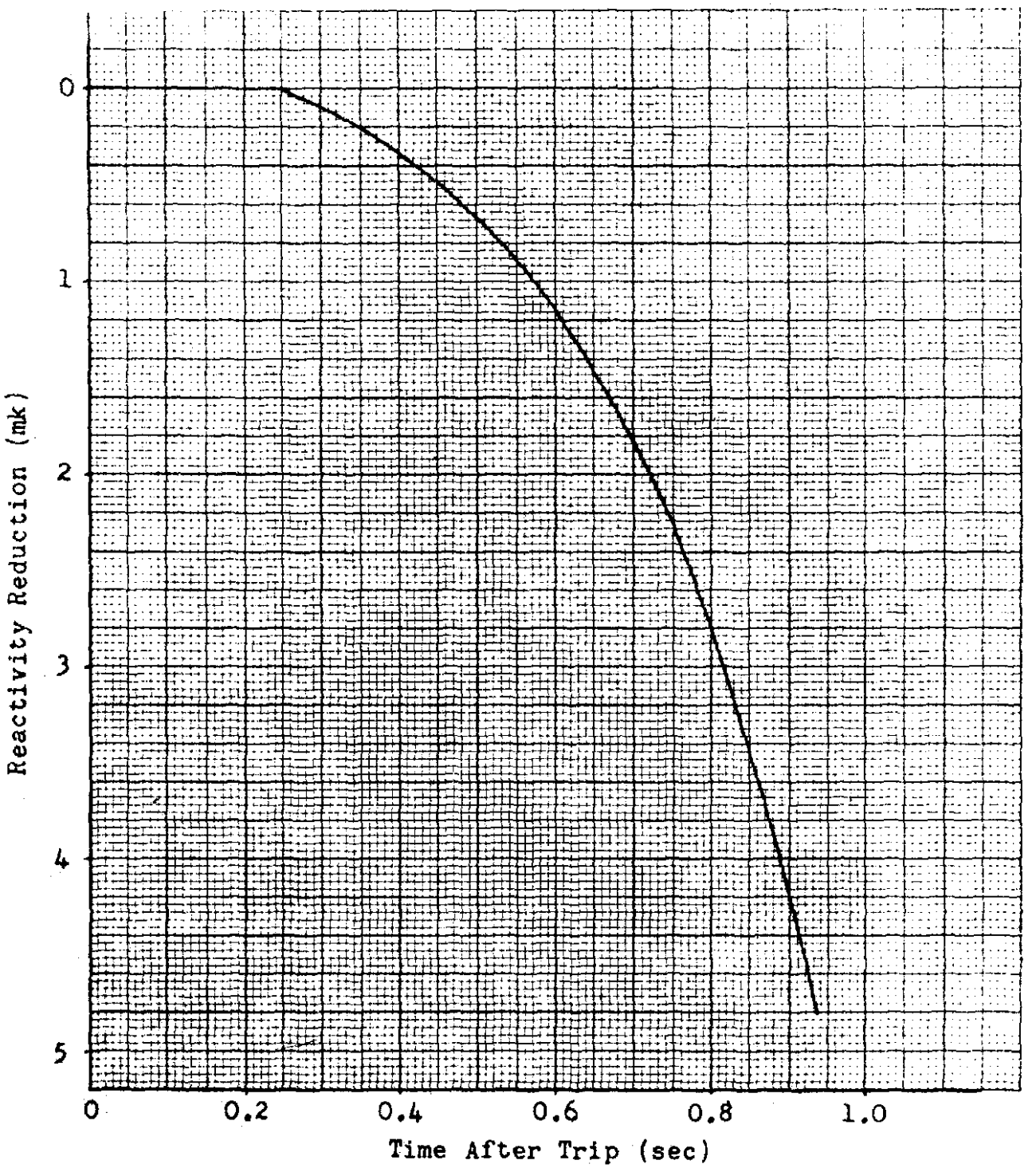

Reactivity Reduction (mk)
0
1
2
3
4
5
0
0.2
0.4
0.6
0.8
1.0
Time After Trip (sec)

Fig. 1

It could be argued that, if such a dumping facility is not provided and the moderator held in the reactor vessel, moderator level regulation cannot be used. In such large reactors, the possibility of xenon oscillations, which are discussed later, make it necessary to use absorber rods for zonal control of the reactor in any case and, therefore, moderator level control as well would appear unnecessary; in which case, fast-acting shutdown or safety absorber rods could just as easily be used. However, moderator dump or partial dump may still be considered as a backup in case of malfunction of some of the shutdown rods.

Nuclear Variables Used for Control

(a) Neutron Power

The thermal power produced in a reactor is measured by the product of the flow rate in the heat transport system and the rise in temperature of the heat transport system across the reactor. However, measurement of neutron power has the advantage of being instantaneous, whereas there are time lags involved in measurement of thermal power.

Neutron power is, therefore, a much more effective and reliable factor for use as a control variable. There are two categories of neutron power measurements, namely:

(1) LINEAR NEUTRON POWER or linear power is the neutron flux or density as measured by an ion chamber and amplified in a linear amplifier. The signal from the amplifier is used in the control system for neutron power regulation, maximum permissible neutron power, and to cause a neutron power trip on overpower (known as a LINEAR N trip) in the protective system.

It may also be used for displaying on meters or recorders. These meters or recorders would normally be calibrated in 0 to 110% of full scale.

Since the range of neutron power to be expected is about 8 decades, the meter or recorder used during manual reactor startup would likely be fitted with an appropriate range switch.

(2) LOGARITHM OF NEUTRON POWER or log power or log N. In this case the ion chamber signal is fed into a logarithmic amplifier and the output from the amplifier displayed on a meter or recorder with a 6-decade scale (ie, 10^{-6} to 1 times full power). Such a scale has the advantage of expanding the low end of the scale. Log N signal could also be used in the protective system as a log N trip.

(b) Reactor Period

If reactor power increases too rapidly it is likely to overshoot the operating power level. This would result in a linear N or log N trip or in excessive fuel temperatures. The regulating system, therefore, limits the reactor period to a value which will avoid such overshoots and the protective system will trip the reactor if the reactor period is excessive. Measurements of reactor period are, therefore, required and these are of two kinds:

(1) Linear Rate - If the linear neutron power signal is fed into an RC circuit, the voltage developed across the resistor is proportional to the rate of change of power (dP/dt). This signal permits high rates to be detected before any significant change has occurred in the actual power level.

(2) Rate of Change of the Logarithm of Neutron Power or rate log - The log power signal is differentiated by feeding it into an RC circuit. The voltage across the resistor is then proportional to d/dt (log P).

$$\text{Since } P = P_o e^{\frac{t}{T}}$$

$$\text{Log}_e P = \log_e P_o + \frac{t}{T}$$

$$\frac{d}{dt} (\log P) = \frac{1}{T}$$

Thus the rate log signal is a measure of the reactor period. It combines the advantages of the expanded scale at low powers and the high rate detection before significant changes in power occur.

ASSIGNMENT

1. What are the three general requirements of a reactor control system?

2. Describe briefly the two functions which a control system must perform to meet the above requirements.

3. Explain the three basic methods by which neutron losses can be changed, or neutron utilization changed, and the methods of control based on these. Indicate how each control method performs both the above functions.

4. Briefly describe the advantages and disadvantages of moderator level control.

5. (a) What is the advantage of linear neutron power measurement over thermal power measurement?

   (b) For what purposes would linear N signals be used?

6. (a) What advantage is to be gained from a logarithm neutron power measurement?

   (b) For what purposes would log N signals be used?

7. (a) What is the meaning of "rate log power" and how is it connected with the reactor period and reactivity?

   (b) For what purposes would rate log signals be used in reactor control?

A. Williams

Nuclear Theory - Course 127

THE APPROACH TO CRITICAL AND THE RAISING OF POWER

There are two stages involved in bringing a reactor up to power and these are:

(a) The approach to critical during which the value of k is increased until the reactor becomes critical.

(b) Increasing the power until the operating level is reached.

The reactor is in its most dangerous condition when it is shut down, for several reasons:

(a) Unexpected increases in reactivity and therefore in power are more likely since it is possible that the regulating and protective system may be "off scale". Under these conditions a fault in either system is not as easy to detect.

(b) Much larger power increases are possible, as a result of reactivity increases, at low power level than at high power levels.

(c) With the heat transport system depressurized, a power surge could result in boiling of the heat transport fluid, which could further increase the reactivity (as will be seen in the next lesson).

(d) The response of the instrumentation is slower compared with the possible reactor periods.

The approach to critical is, therefore, a procedure that must be undertaken with a great deal of respect and it will be considered at some length in this lesson.

Subcritical Operation

During the approach to critical the reactor is subcritical and so, before considering the approach itself, it would be desirable to have considered the manner in which a reactor behaves when it is subcritical and k is less than 1.

It was shown, in a previous lesson, that if a source of neutrons, of strength P_s watts, exists in the reactor, then, on shutdown, the reactor power will decrease to a value given by:

$$P = \frac{-P_s}{\delta k}$$

where δk is the amount of negative reactivity introduced when the reactor is shut down.

Now the reactivity is a measure of how far the reactor is from being critical and $\delta k = k - 1$ or $-(1 - k)$.

$$\text{Therefore} \quad P = \frac{P_s}{-(1-k)} = \frac{P_s}{1-k} = \frac{1000\ P_s}{\text{milli-k below critical}}$$

If P/P_s is plotted against time for various values of k, a series of curves, as shown in Fig. 1, are obtained. Of what significance are these curves?

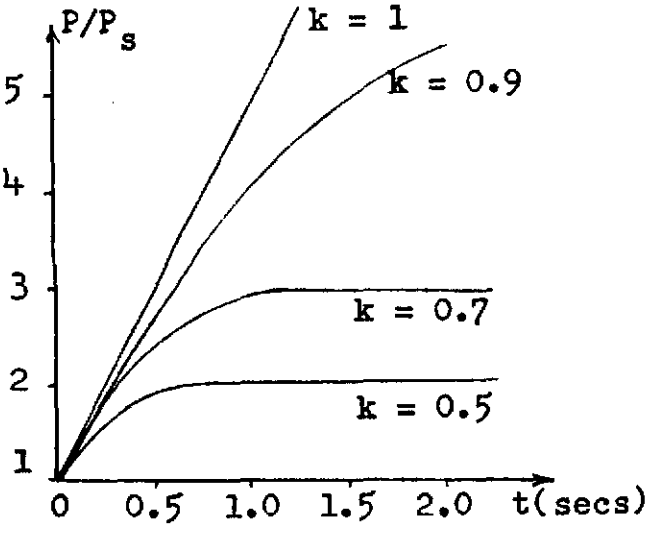


Fig. 1

Supposing that the value of k was initially negligible. If now the moderator level was raised until k = 0.5, the reactor power will increase as shown in the lower curve and level off at 2 P_s in half a second or so.

As k is increased, the power levels off more slowly but at a higher value, until, when k = 1 the power continues to increase without levelling off at all.

Now, if P_s is very small, as it is when spontaneous fissions only supply the neutron source, the power at which the reactor levels out is very small until k is close to 1.

Suppose P_s = 0.0001 watts with spontaneous fissions.

$$\text{When } k = 0.1 \quad P = \frac{P_s}{0.9} = \frac{0.0001}{0.9} = 0.00011 \text{ watts}$$

which would be too low to get much of a reading even on sensitive fission count rate meters.

$$\text{With } k = 0.5 \quad P = \frac{0.0001}{0.5} = 0.0002 \text{ watts}$$

Therefore k would have to be close to 1 and the reactor close to being critical before even count rate instruments were effective. In addition the neutron multiplication is subject

to wide statistical fluctuations. Since it is necessary, at each value of k, for the power to level off as in the curves in Fig. 1, to determine whether or not criticality has been achieved, the critical point may well be passed while the neutron power is still low. By the time the power has risen to the point where it becomes obvious that it is increasing, the rate of rise may be so rapid that the reactor cannot be shut down fast enough to prevent damage.

To avoid this possibility a neutron source is placed in the reactor so as to artificially increase P_s and provide a good count rate. When k is large enough for a good count rate, with spontaneous fission sources alone, the neutron source is removed.

With photoneutron sources, on the other hand, P_s may be 30 watts.

Thus when k is only 0.1 or 900 mk below critical $P = \frac{30}{0.9}$

$= 33.3$ watts.

This value is only about 7 decades below full power and within the range of the most sensitive linear neutron instruments which normally go down 8 decades. Since this source term is so much greater than that due to spontaneous fission, special fission chambers and count rate meters would not be required.

The First Approach to Critical

In the CANDU type of reactor, the approach to critical is likely to be made by raising the moderator level until enough fuel is covered to sustain a chain reaction. If absorber rods are also available for reactivity adjustment, they would be completely withdrawn. If variation in moderator level is not possible and reactivity control is by absorber rods only, then the approach to critical would have to be made by gradual withdrawal of the absorber rod. However, it will be assumed that the former method is to be used, as in fact it was at NPD and will be at Douglas Point.

The initial critical level, with fresh fuel, will be much lower than the normal operating level. Some attempt is usually made to raise this initial critical level such as by replacing some of the normal natural uranium or uranium oxide fuel with depleted fuel (ie, having less U-235 content than natural uranium). However, even with depleted fuel the initial critical level in NPD was 97.5" compared with the normal operating level of about 160". The estimated initial critical level in Douglas Point is only 36% of full calandria and this would not cover the depleted fuel region. Also a 1" change in level in this region is equivalent to a 2.5 mk reactivity change. Boron addition to the

moderator is being considered which would raise the initial critical level to 50% of full calandria. However, this involves the control of an additional parameter at a time when this is least desirable.

The first approach to critical is considered more hazardous than subsequent approaches because:

(a) There are only spontaneous fission sources and no photoneutron sources, in the core and therefore the normal neutron power instruments are not nearly sensitive enough to measure the neutron density.

(b) The neutron power instruments have not, in any case, been calibrated and, therefore, cannot be relied on.

(c) The automatic regulating system is inoperative.

(d) The critical height of the moderator is not known, ie, it is not known at what moderator level the reactor will go critical.

(e) Large reactivity increases are possible because the xenon and other fission product poison are absent.

The first approach to critical is, therefore, carried out in such a manner that recognition is given and allowances made for the problems that do exist. Because the regular neutron power instruments cannot be used, sensitive neutron fission chambers or BF_3 counters are used during the approach. These fission chambers or counters are normally lowered into the core so that they are in a higher flux region. They are connected to count rate meters, because the actual power level is so low, so that the neutron count rate is measured rather than power. Three such detectors are normally used so that three independent count rates are established. This allows for error or failure of one counter and also allows for greater flexibility when the neutron density increases. A neutron source is placed in the core so that a reasonable count rate is obtained at values of k well below critical. This avoids statistical errors on the counters.

If the detector used is sensitive enough to be used with spontaneous fission sources only, a neutron source would still be used to check the operation of the detectors.

The moderator level is raised in small steps (1 or 2 inches at a time) and the neutron power allowed time to level out as shown on the curves in Fig. 1. For small values of k the count rate levels out rapidly but, as k increases, it takes progressively longer to level out. As k increases the count rate increases and eventually the fission source can be removed. Also the fission chambers have to be moved further out from the core

to prevent them saturating. The critical moderator level is predicted by plotting the reciprocal of the count rate against the reciprocal of buckling, $1/B^2$, on the three independent counters. For a cylinder:

$$B^2 = \frac{(2.405)^2}{R^2} + \frac{\pi^2}{H^2}$$

Therefore, for a vertical cylindrical reactor, where R remains constant and only H changes, the reciprocal of the count rate is plotted against H^2.

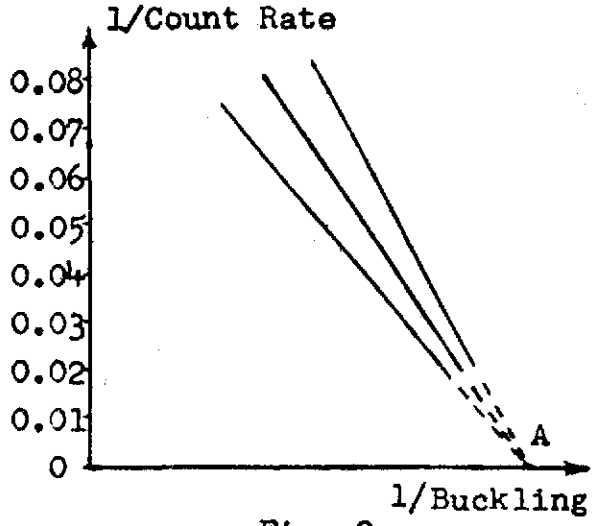


Fig. 2

For a cylindrical reactor, with its axis horizontal, the portion of the core covered by moderator, is only a portion of the cylinder. Therefore, the buckling is a more complex expression than that shown above. A curve, relating buckling to the distance L of the moderator level below and above the reactor centre line, is shown in Fig. 3. The reciprocal of the count rate is plotted against the value of $1/B^2$ for each value of moderator height.

The three straight lines shown in Fig. 2 are then obtained, which converge on the point A when the count rate is very large or the reactor is critical. The point A, then, gives the value of $1/B^2$ when the reactor will be critical and this can be converted back into the critical moderator height.

Because of the difficulty of estimating accurate values for the buckling for a horizontal cylinder, an alternative approach is sometimes used. As was shown previously, the multiplication factor of source neutrons in a subcritical reactor is $1/(1-k)$. Thus the neutron count resulting from the neutron source is enhanced by a factor of $1/(1-k)$.

$$\text{ie,} \quad \text{Count Rate} = P_s \times \frac{1}{(1-k)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Therefore a graph of the reciprocal of the count rate against $(1-k)$ should be a straight line and so should the graph of reciprocal count rate against k. The multiplication factor k can be calculated as a function of the moderator level. When the reciprocal count rate is plotted against k, three straight lines are obtained as in Fig. 2 and the point A again extrapolated. The value of k at A is then converted back into the critical moderator height. This approach was adopted at NPD and is likely

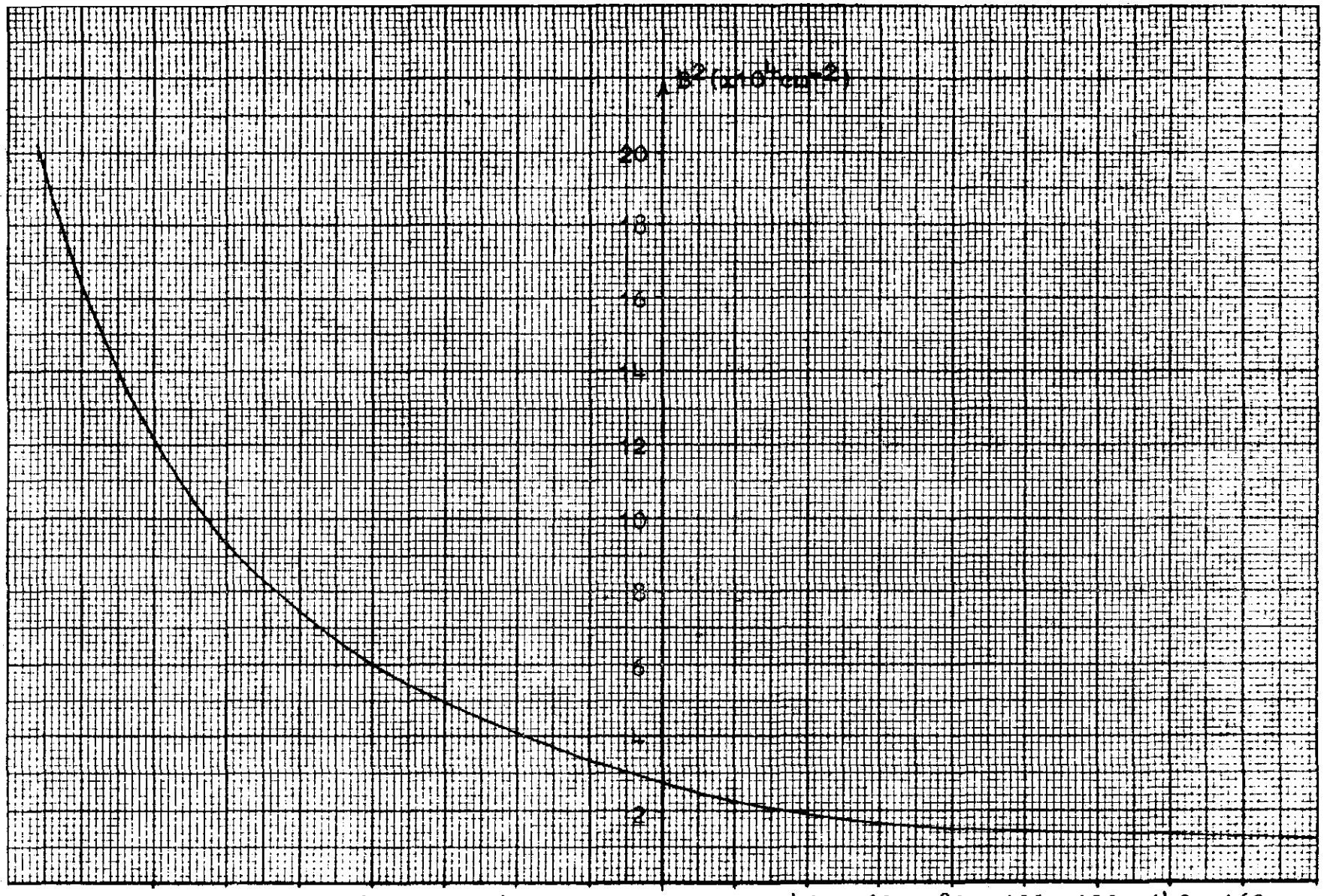

B² (x10⁵ cm⁻²)
20
18
16
14
12
10
8
6
4
2
-160 -140 -120 -100 -80 -60 -40 -20 0 20 40 60 80 100 120 140 160
Distance below reactor centre line (cm) Distance above reactor centre line (cm)

Fig. 3

to be the method used at Douglas Point. Fig. 4 shows a typical plot on one of the fission chambers during the NPD first approach to critical. Slight deviations from the straight line can be observed as the moderator level passes the various rows of fuel.

When the critical height has been established, as accurately as possible, it is approached cautiously until the power starts to increase on a long period, which shows that the critical moderator level has just been exceeded. For instance, in NPD when the moderator level was within 1.5 inches of the projected critical height, the level was raised through critical and the reactor power allowed to increase on a 150-second period. At 10^4 counts per second the level was adjusted to hold the count rate steady. This adjusted level is then the initial critical level. Note that prior to this last step being taken, one detector is moved away from the maximum flux region to ensure that one chamber continues to read in the range in which it had been checked and calibrated.

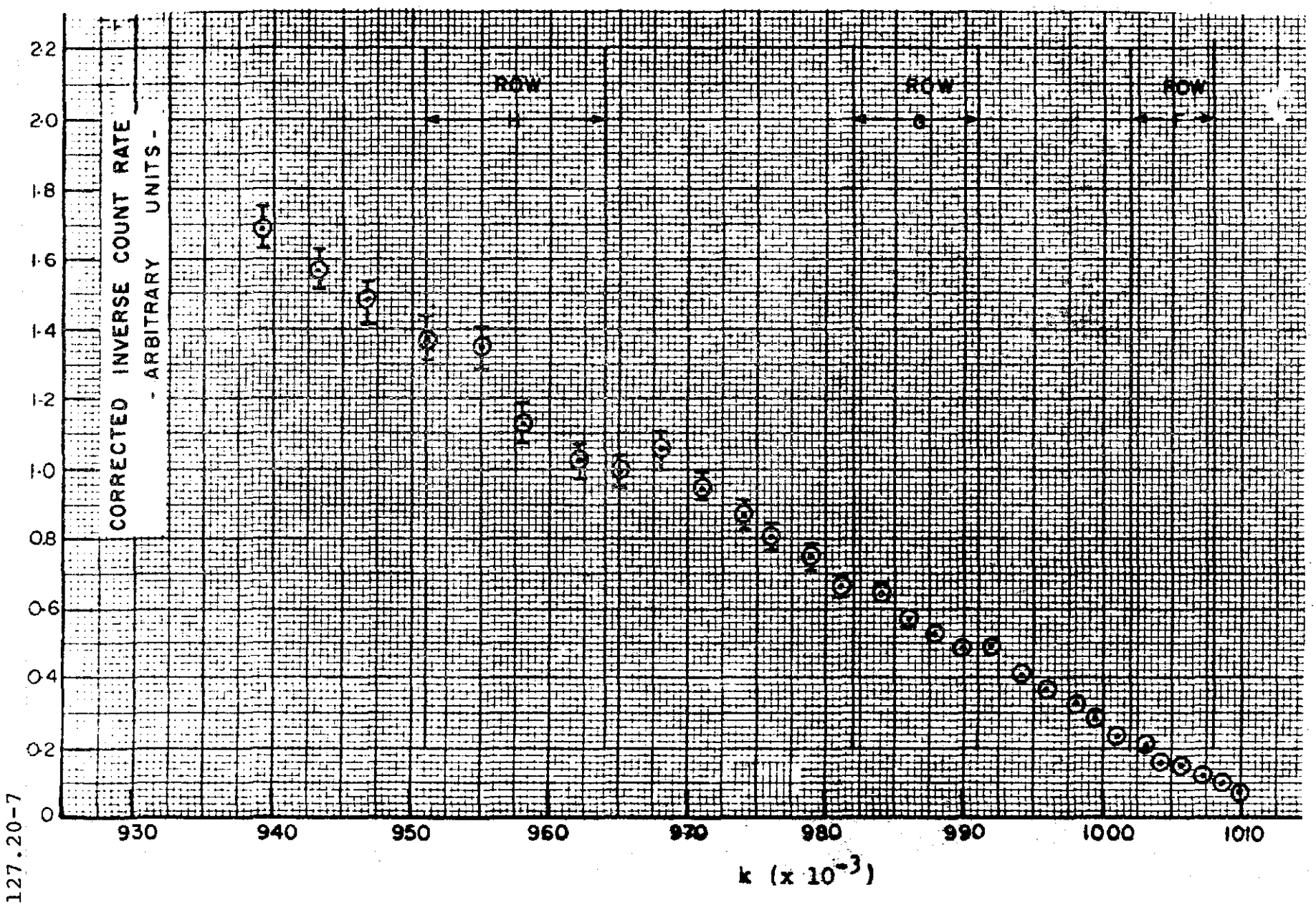
The same approach to critical, as has just been described, would have to be used if the reactor is shut down for longer than four months or so. The fission products would then have decayed to such an extent that the photoneutron sources are too small to be of value.

In a graphite or light water moderated reactor every approach to critical is the same as the initial approach since no photoneutron sources are built up.

Subsequent Approaches to Critical

In a heavy water moderated reactor, fission products accumulate in the fuel if the reactor has been operated at power. So, for subsequent startups, the photoneutron source in the reactor will be large enough to give a reading on the lower range of the most sensitive neutron power instrument. Therefore, no special neutron counters or neutron sources are required. However, the neutron power signal is too weak for satisfactory automatic operation of the regulating system, unless the shutdown was of short duration. The approach to critical is, therefore, made on manual control using the normal neutron power instruments.

The moderator level is allowed to rise in steps of 1" or so at a time. After each 1" step the moderator level is held constant until the subcritical power reaches an equilibrium value. When the power continues to increase the critical level has been exceeded. The level is then adjusted down until the power is held constant. An estimate of the critical level can be made by plotting the reciprocal of neutron power against the moderator height. The point where the graph crosses the moderator height axis will indicate the possible critical level.


CORRECTED INVERSE COUNT RATE
- ARBITRARY UNITS -
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0
930
940
950
960
970
980
990
1000
1010
k (x 10⁻³)
ROW
ROW
ROW

127.20-7

Fig. 4

An alternative approach is to allow the moderator level to rise slowly but steadily initially. An estimate is then made of the increase in moderator height required to double the subcritical power. Half this increase in height should then be required for a further doubling of the power or halving the δk value below critical since:

$$P = \frac{P_s}{\delta k} \text{ numerically}$$

and so when δk is halved, P is doubled. An assessment can then be made by repeating this process, of what the critical level will be. The increase in moderator height required to double the power is rechecked as the level rises until the level is an inch or two below critical. The critical point is then approached cautiously until the power is seen to rise continuously on a long period. The rate log meters should also be indicating the reactor period.

It may not, of course, be considered necessary to go critical on manual control. It is only really necessary to allow the moderator level to rise until the neutron signal is strong enough for automatic regulation. A switch-over is then made from manual to automatic control and the regulating system allowed to bring the reactor to critical.

Changing Reactor Power

Once the reactor is critical, it may be kept at any power level by adjusting the moderator level or control rod positions to keep $k = 1$ or $\delta k = 0$. If the power has to be increased, the moderator level is raised or control rod moved out of the reactor to make k just greater than unity, or δk slightly positive.

Fig. 5, on the following page, shows such a movement in moderator level or control rods at A.

The figure also shows the corresponding changes in k and δk and the resulting exponential increase in power. The reactivity change possible is usually limited by design so that the reactor period during the power increase is long. There is also a reactor trip if this period becomes too small, ie, the rate of increase of power is too high.

At B, the required power level has been reached and the moderator level or control rods are returned to the point where $k = 1$ and $\delta k = 0$. This is somewhat different from the control method in conventional power plants.

In a conventional plant, when an increase in the thermal power is required the firing rate is boosted by opening the fuel

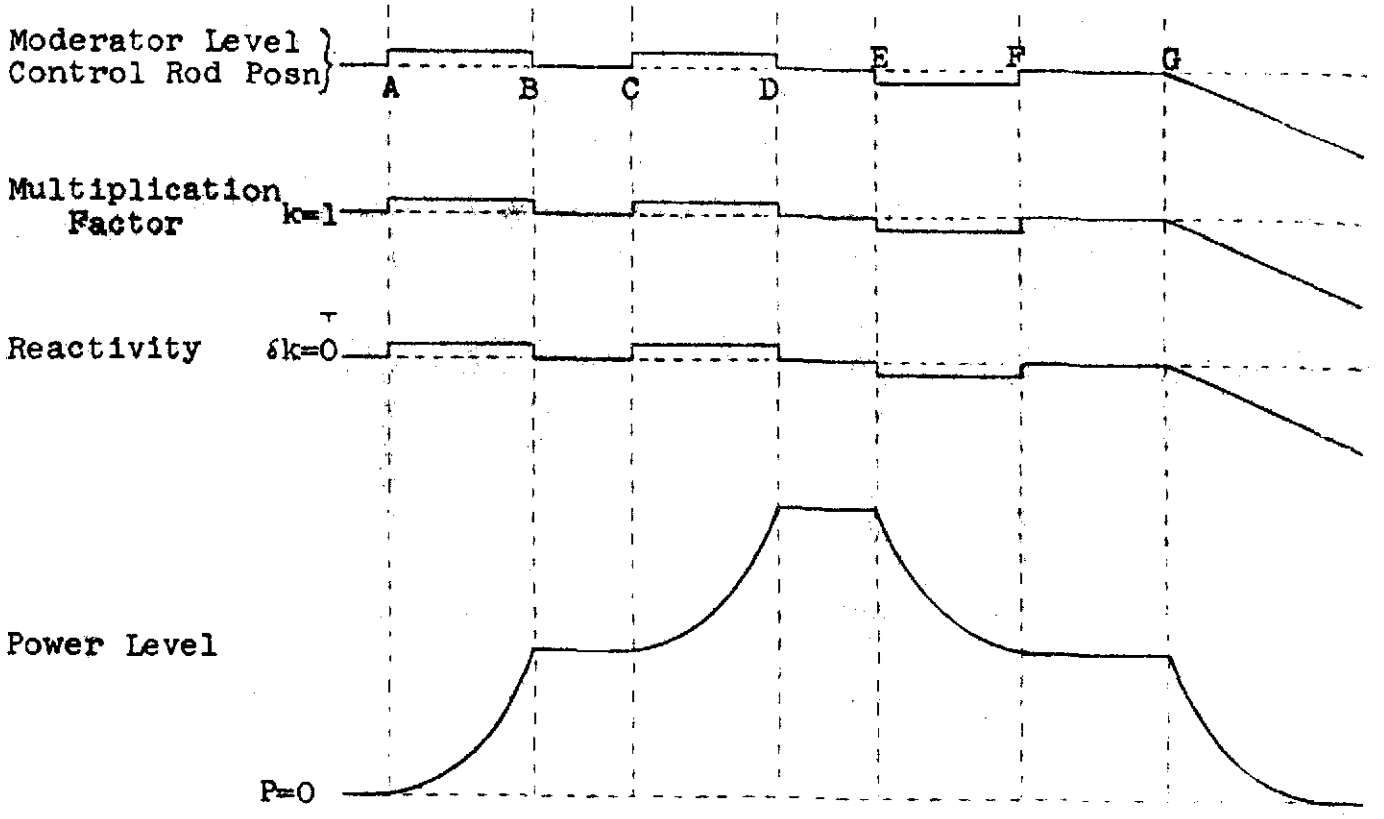


Fig. 5

valve wider and increasing the air flow. The valve is then left open at the new setting. However, in nuclear plants, the moderator level is only raised or the control rods moved while the power is being raised by increasing neutron density. When the correct power level is reached, the moderator or control rods are returned to the old level to prevent further increase in neutron density.

If the power remained steady from D onwards, there would be a gradual rise in moderator level or a gradual movement of control rods out of the core to compensate for fuel burnup or poison buildup, so as to keep $k = 1$ or $\delta k = 0$.

At E the moderator level is lowered or the control rods moved further into the core to lower k slightly below unity. This causes the reactor power to decrease exponentially until the desired lower power level is reached at F. The moderator level or control rod positions are then adjusted so that the reactor is again just critical. At G is shown the beginning of a reactor shutdown controlled by the regulating system. The reactivity decreases slowly and the reduction in neutron power is not rapid compared to the corresponding changes following a trip.

ASSIGNMENT

1. In what condition or state is a reactor in its most dangerous state and why?

2. In terms of subcritical reactor operation, why is it necessary to place a neutron source in the reactor core during the first approach to critical?

3. (a) Give the reasons why the first approach to critical is more hazardous than subsequent approaches to critical with D_2O moderated reactors.

   (b) What additional instrumentation would be required for the initial approach to critical and why?

4. (a) How is the critical moderator level predicted from the count rate?

   (b) How is it known when the reactor is critical?

   (c) The following count rates were recorded, at the moderator depth indicated, during a first approach to critical with a cylindrical reactor, the axis of which is horizontal. Using the graph in Fig. 3 of the lesson to obtain values of the buckling, determine the critical height of moderator. The depth of D_2O at the centre line is 224 cm.

Depth of D_2O (cm)	53	77	117	150	178	203	216	228	238
Count Rate (cpm)	55.5	59.2	68	84	101	141	185	333	606

   (d) What alternative approach is adopted to predict the critical level if the buckling for the reactor in (c) is too difficult to calculate?

5. (a) In a D_2O moderated reactor, why are fission counters not required during subsequent approaches to critical?

   (b) Why is it still necessary to make the initial part of the approach to critical on manual control?

   (c) Briefly describe the two alternative methods that can be used for the approach to critical on manual control and the methods of predicting the critical level in each case.

   (d) Why might it not be necessary to go all the way to critical on manual control and what would be the alternative procedure?

A. Williams

Nuclear Theory - Course 127

EXAMPLES OF PRACTICAL REACTOR BEHAVIOUR

The effects of poison buildup, temperature changes and fuel burnup have been considered, in previous lessons, as separate effects. However, since fission product poisons, temperature and burnup of fissile material change with changes in reactor power, considerations of practical reactor behaviour must involve all of these effects. The manner in which a reactor responds to power changes will also be determined by all these factors. The examples of reactor behaviour, given in this lesson, will illustrate how these factors affect the reactor behaviour.

Some examples of reactor behaviour, given in the Level 2 Nuclear Theory course will be reviewed in order to present a complete picture and maintain continuity.

Effect of Xenon Delay on the Approach to Full Power

On reactor startup, following a shutdown, the xenon load is zero or very small. If absorber rods are used as the only control elements, the calandria will always be full of moderator and the increase in reactivity will be compensated for by insertion of absorber rods. No restriction is thus placed on the maximum permissible power.

However, if moderator level is used as a control element, the critical moderator level is much lower than when equilibrium poison has been established. The resulting thermal neutron flux distribution and the fact that only a part of the normal reactor core is being used, limits the power to a value well below full power. The variation of permissible power with moderator level, in NPD, is shown in Fig. 1. This delay in reaching full power is known as XENON DELAY. It can only be avoided by introducing neutron absorbing material into the core in order to operate the reactor with a higher critical moderator level. Liquid poison, in the form of boric acid injected into the moderator, is proposed for Douglas Point.

If xenon delay does limit the operating power, as at NPD, the manner in which the reactor power is increased and the changes in moderator level that occur are shown in Fig. 2.

At A, reactor power is increased to a nominal 5% of full power to B, to warm up the heat transport system and to raise steam pressure. The reactor power remains constant from B to C until the turbine is up to speed and the generator synchronized to the system.

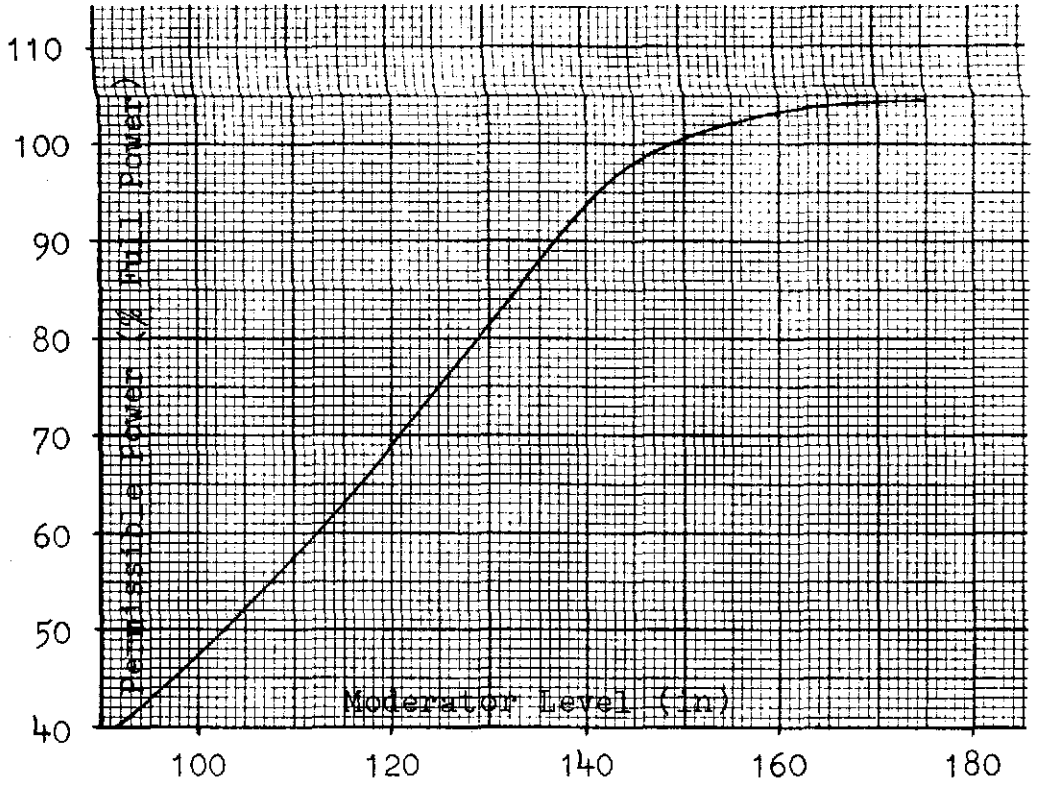

110
100
90
80
70
60
50
40
Permissible Power (% Full Power)
Moderator Level (in)
100
120
140
160
180

Fig. 1

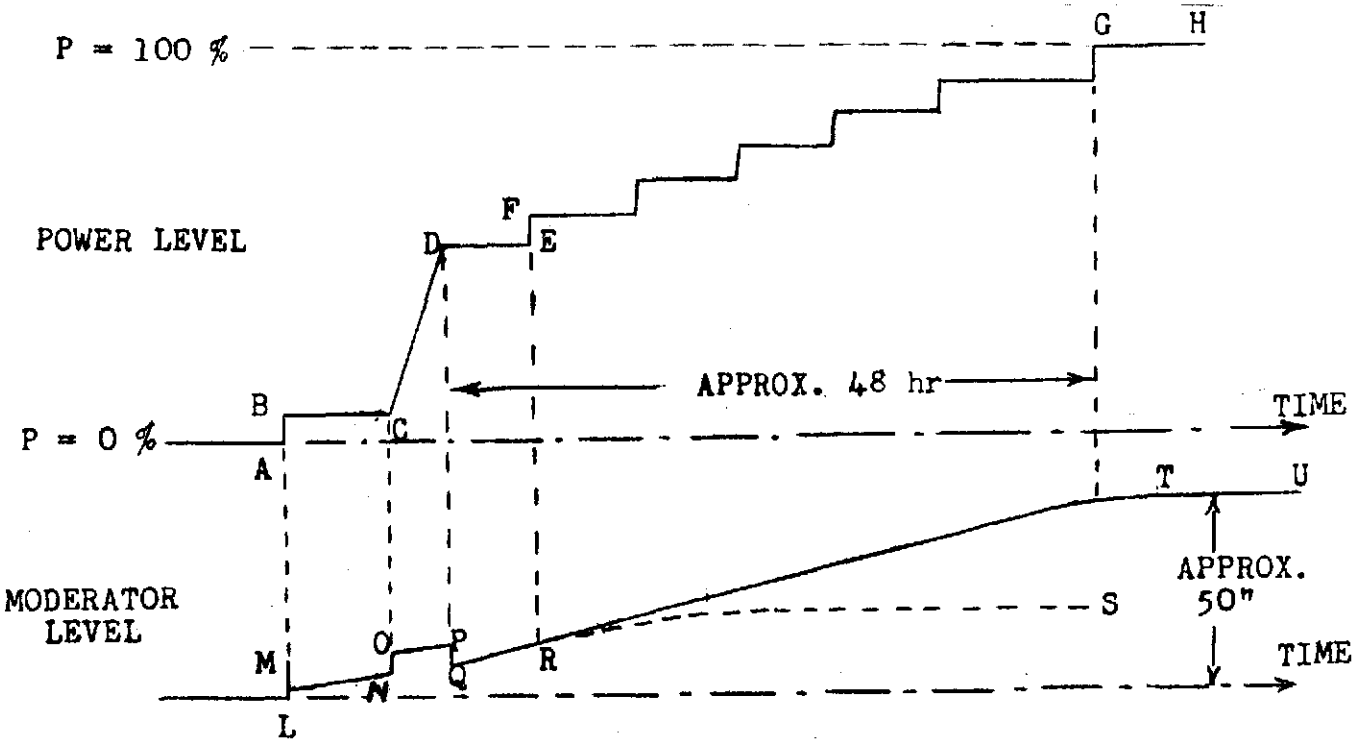

P = 100 %
P = 0 %
POWER LEVEL
MODERATOR LEVEL
A
B
C
D
E
F
G
H
L
M
N
O
P
Q
R
S
T
U
APPROX. 48 hr
TIME
TIME
APPROX. 50"

Fig. 2

At L the moderator level rises to M long enough for the power increase AB to take place. The level then drops back to maintain the reactor just critical. The new critical level is slightly above the initial critical level because of the power coefficient. It then rises to N due to the loss of reactivity associated with the heat transport system negative temperature coefficient.

The load is increased at C and the reactor power rises to about 50% of full power. The moderator level rises and remains above critical along OP during the power increase. When the power is levelled off at D the moderator level drops back to the critical level at Q. If the power remained at D the moderator would follow the curve QRS as the xenon built up to equilibrium.

In order to reach full power as quickly as possible, the power is raised in steps such as EF whenever the moderator level, and hence the permissible power, is high enough to allow it. This keeps the xenon buildup continuing until it reaches its equilibrium TU slightly after the power reaches 100% at GH. The fine structure on the moderator curve associated with each individual power step has been ignored for simplicity.

Moderator Level Response to Load Changes

Although moderator level response to load changes is considered here and, therefore, a moderator level control element is assumed, the response of absorber rods would follow similar patterns. Where the moderator level is shown to rise, absorber rods would move out of the reactor. A fall in moderator level corresponds to a movement of absorber rods into the reactor. Five types of load changes are considered:

(a) Minor Load Changes

A minor change in load may occur due to some change in setting in the regulating system which changes the steam pressure by a few psi.

Fig. 3 (a) shows reactor changes following such a minor load change and restoration of full load. Fig. 3 (b) shows the response of the moderator level when fuel burnup is ignored and Fig. 3 (c) shows the moderator level response when the effect of fuel burnup is not to be ignored.

The reactor power remains steady up to A, when a small decrease in power, AB, occurs due to a load change. Ignoring fuel burnup the level remains constant along EF when a temporary drop occurs to G to provide the necessary negative reactivity. When fuel burnup is taken into account the level rises from M to N and then drops to O. In both cases a rise will then occur due

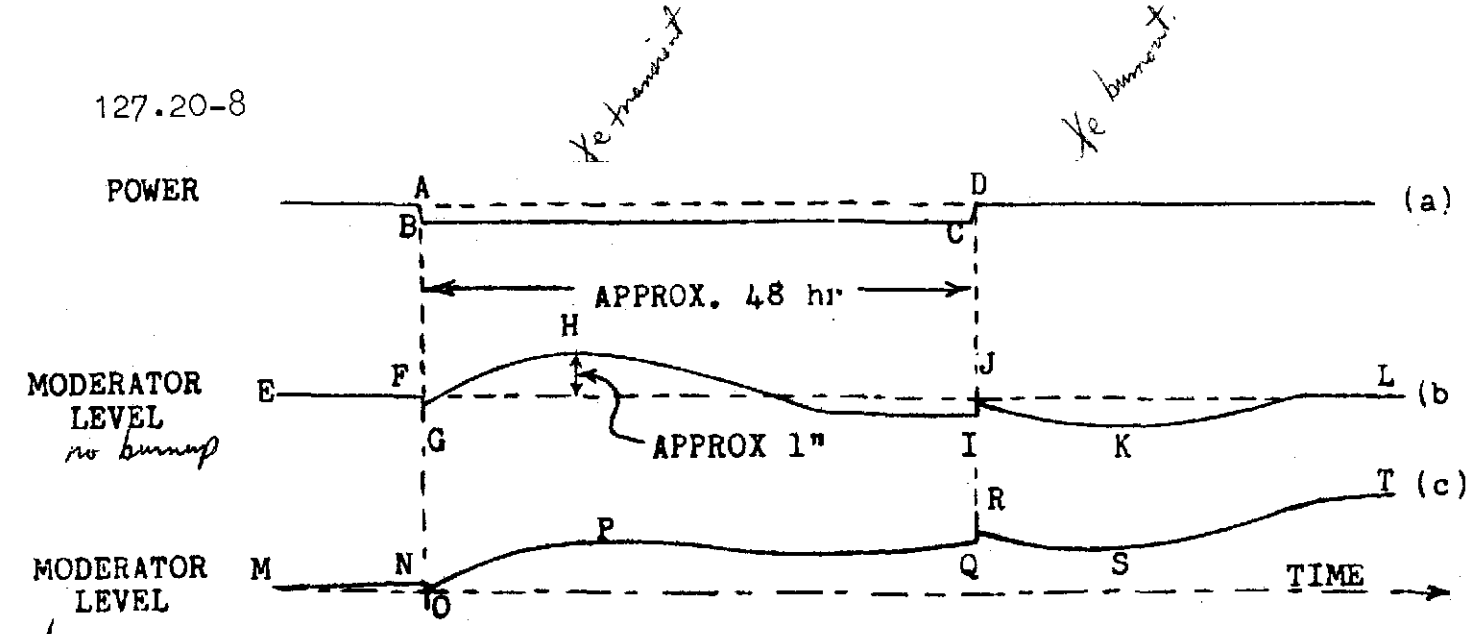


Fig. 3

to the xenon transient. The xenon will eventually come to an equilibrium which is lower than the original amount and the moderator level is steady at I. In curve (c) the fuel burnup effect may be large enough so that there is no decrease in level from P to Q.

When the power is restored from C to D there is a transient rise in moderator level to J (or R) to supply the necessary positive reactivity during the power change. The level then drops back to critical and a downward xenon transient starts. This transient is caused by an increase in xenon burnout which temporarily makes xenon destruction (burnout and decay) greater than xenon production. The production rate builds up however and the level eventually returns to L, the original critical level, if burnup is ignored. If fuel burnup is allowed for, the final moderator level will be at T (curve (c)).

(b) Larger Load Changes

This is the load change that takes place when, for instance, a turbine emergency stop valve is tested by closing it. The reactor power change that occurs may be as much as 15% and the resulting xenon change is, therefore, much harsher than in case (a).

Fig. 4 shows how the moderator level would respond to such a reduction in power followed by a restoration of full power.

The load decrease starts at A and continues to B where one stop valve would be fully shut. The load then returns as the valve is opened from B to C. Fig. 4 (b) shows the moderator level if xenon and fuel burnup are ignored. The level decreases from E to F with some overshoot but remains subcritical until G. Positive reactivity is then required and the level rises to H (again with some overshoot) and remains high

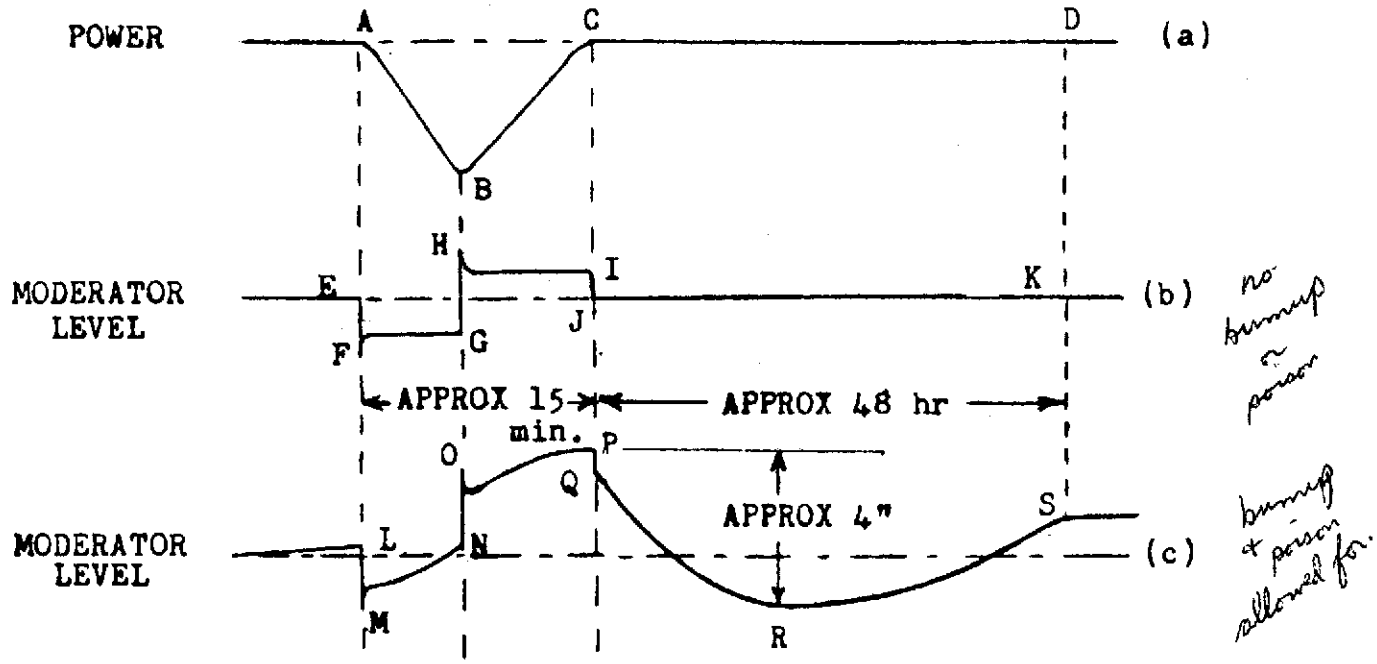


Fig. 4

enough to keep the reactor supercritical while the power is increasing. The level returns from I to J, the original critical level, when the power stops rising.

In Fig. 4 (c) the effects of fuel burnup and xenon have been included to show what actually happens. The shape of the curve from L to Q is modified from (b) mainly by xenon buildup which is most pronounced when the power is lowest (at B in (a)). The transient QRS is due to xenon burnout and is similar to the transient in Fig. 3. The final level S is higher than L due to fuel burnup.

(c) Substantial or Complete Load Rejection

This is the type of load rejection that occurs when a loss of line occurs and it is basically an exaggereated form of the load reduction in (b). A partial load rejection may occur very rpaidly or, in the extreme case, all the load except the station service may be rejected. The changes in moderator level, that would follow a sudden complete rejection of external load, are shown in Fig. 5. The load is rejected at A and the reactor power decreases to about 10% of full power at B. There is a sudden and substantial drop in moderator level, from D to E, to provide the necessary decrease in reactivity. If the load rejection continues, the moderator

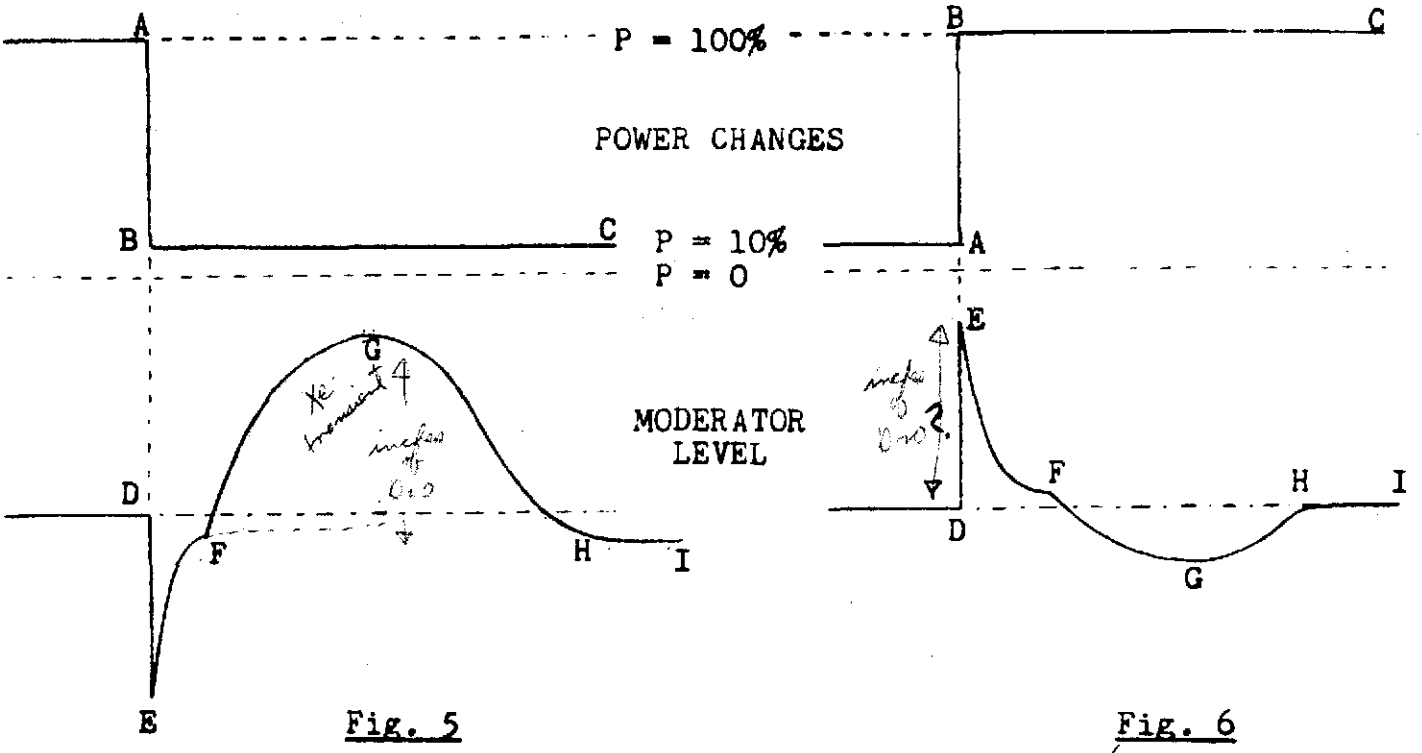


Fig. 5 (c)

Fig. 6 (d)

level would recover along EF, except for the xenon transient that occurs because of the reduction in power. Because of the xenon transient, the moderator level rises along FG and then decreases along GH, because of the subsequent xenon decay. The moderator level finally settles down at I, at a lower value than D, because of the smaller xenon load. The substantial increase in moderator level along FG will cause the reactor to poison out, unless load is restored quickly.

(d) Substantial Sudden Load Demand

In a base load generating station, already operating at or close to 100% power, sudden load demands do not normally occur. They only occur after a load rejection when the line is restored. Fig. 6 shows the moderator level changes that would occur following restoration of full load from, say 10% load. Again the reactor power changes rapidly from A to B and a sharp moderator rise from D to E occurs. The moderator level then recovers along EF. However, xenon is being burnt out faster than it is being produced, at the higher power, and so the moderator level continues to fall along FG. The rate of production of xenon eventually exceeds the rate at which it is removed and the moderator level rises along GH, and settles down at the previous level I.

(e) Load Rejection Quickly Followed by Load Restoration

As was suggested in (d), a load rejection due to loss of line may be followed quickly by a sudden load demand when the line is restored. Sudden changes are shown in Fig. 7.

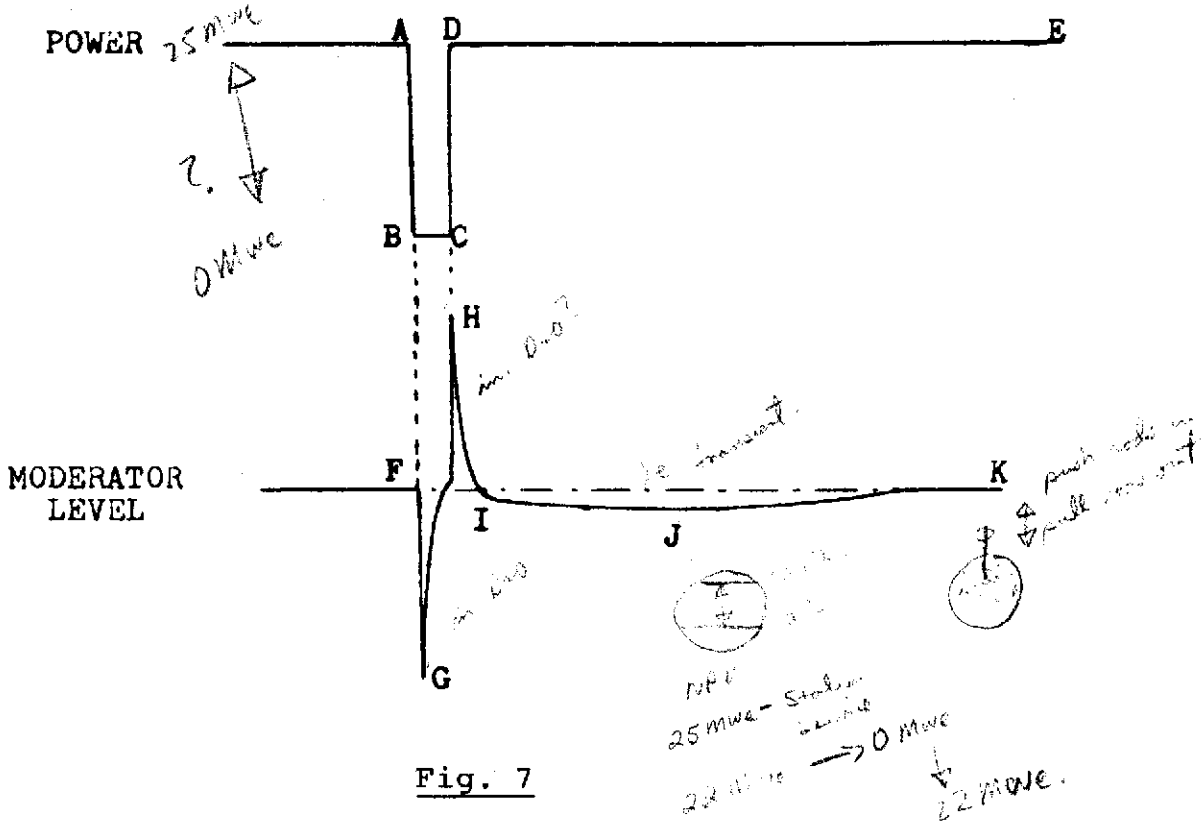


Fig. 7

A rapid decrease in power, from A to B, is caused by the load rejection and, as in (c), the moderator level drops from F to G. Very shortly after this the line is restored and the level rises above critical to H. When the power has been restored to 100% the moderator level returns to the critical value and goes through a xenon transient as in previous cases.

Note that the effects of fuel burnup have been ignored in (c) (d) and (e).

ASSIGNMENT

1. When reactor power is raised, the first increase in power is to nominal 5% of full power.

   (a) How long is reactor power kept at this value?

1. (b) What moderator level changes occur during this period and why?

2. Following this period at 5% full power, reactor power is further increased because of load demand.

   (a) What limit is there on this second increase in power?

   (b) Why should this limit on reactor power exist?

   (c) Why can this limit be exceeded later?

3. After the limit in 2 (a) can be exceeded, how is reactor full power achieved and what basic principle is involved in this method of achieving full power?

4. (a) How does the initial drop in moderator level, following a minor load reduction, differ from that following a major load decrease or rejection?

   (b) How does the subsequent rise in moderator level differ and how might this affect the reactor if the load is not restored quickly?

   (c) Why, in the case of load reduction, is the final moderator level lower than the initial level before the reduction?

5. (a) When is a sudden substantial load increase or demand most likely to occur?

   (b) In general, how does such a load increase modify the moderator level response in 4 (b), when it occurs as in 5 (a)?

   (c) How does the rate at which such a load increases affect moderator level response?

A. Williams

Nuclear Theory - Course 127

REACTOR STABILITY

Many aspects of reactor theory have been considered, particularly those effects or changes which cause changes in reactivity, eg, xenon buildup and temperature coefficients.

Two factors which can affect the inherent stability of a reactor are temperature changes and xenon poisoning.

Temperature and Reactor Stability

When a study is made of the stability of a reactor, the regulating system and its response to various conditions in the reactor is normally included. The reactor itself may be more or less inherently stable. However, depending on its temperature coefficients, if an overall power or temperature coefficient is negative, this will help to prevent power transients. (This was demonstrated at NRX as discussed in the earlier lesson on the effects of temperature changes.)

In addition to the large relatively slow transient, it is also possible to have the reactor power oscillate with a high frequency due to various time delays in the reactor and the regulating system. Time delays can be associated with:

(1) The response time of the regulating system and control method.

(2) The time associated with flux changes, ie, there is a small interval of time between a flux disturbance occurring in one part of the core and the change in flux being detected by ion chambers in another part of the core.

(3) The materials in the core (such as moderator) have an appreciable heat capacity. This results in a time lag between a change in power and the resultant temperature change.

If the system stability is good, any changes in power which tend to start an oscillation are quickly damped out. In this respect, the delayed neutrons are important since they tend to slow down any change in reactor power.

Because of the time delay associated with the heat capacity of the core material, the fuel temperature coefficient or power coefficient may have a more important influence on the reactor stability than the other coefficients. A reactor may have an overall positive temperature coefficient and still be inherently stable, provided that the fuel temperature coefficient is negative. Even though the positive moderator temperature coefficient is greater than the negative fuel temperature coefficient, the response time of the fuel is considerably less than that of the moderator. Thus, when an increase takes place in the power, the fuel temperature rises soon after the power increase but moderator temperature rise will be delayed because of its large heat capacity and moderate thermal conductivity. Hence, a transient increase in fuel temperature will be counteracted promptly by the effect of the negative temperature coefficient long before the positive moderator coefficient can have an effect. The reactor would, therefore, be inherently stable against transient temperature changes.

Xenon Oscillations

When xenon and samarium buildup was considered, it was assumed that the poisoning and the reactivity load applied to the reactor as a whole. No account was taken of the possibility of localized changes in xenon poisoning which can have a very important effect on reactor stability.

Suppose, for the sake of argument, that the automatic control system is "frozen" and unable to change reactivity one way or the other when the xenon poison has reached equilibrium concentration. Now suppose that a small decrease in flux occurs. This will decrease the rate of removal of xenon without appreciably changing its rate of production. The xenon concentration increases and, since the control system is unable to counteract the xenon load, the reactor becomes subcritical. The flux decreases further, there is a further increase in xenon, and the reactor "poisons out".

Many hours later, after all the iodine has gone and the xenon has decayed, there will be enough reactivity for the reactor to become critical, since the control system is still frozen. The flux will increase and more xenon will be removed and the reactor becomes supercritical. It will remain supercritical until the xenon builds up sufficiently for it to become subcritical once more. So the flux and power will oscillate, and the effect is known as XENON OSCILLATIONS.

The above considerations suppose that the regulating system remains frozen whereas, normally, the regulating system counteracts any flux disturbance as soon as it takes place, and thereby keeps the reactor power constant. However, a local disturbance

can be set up in a reactor by on-power refuelling, for example. Refuelling causes a sudden localized lowering of xenon concentration which, in turn, causes the flux to increase in this locality. The above cycle is, then, initiated unless corrected by the regulating system. However, corrective measures by the regulating system may not be effective if the reactor is so large that different regions in the reactor function as independent units.

If the neutrons produced in one region of the reactor do not cause significant fissions in another region, then the two regions can act independently of one another. The criterion that determines whether or not this is possible is the degree of neutron leakage from the one region to the other. In a reactor such as NPD the core is small enough to permit a disturbance started in one region to have an effect in another region. The xenon and flux changes would therefore affect the whole core and a regulating system based on flux measurements in one locality can correct the flux disturbance and prevent xenon oscillations from being initiated.

If the reactor is large, or if different regions of the core are separated by a region of high neutron absorption, leakage of neutrons between regions is very small. A disturbance started in one region has little effect in another region. Thus, if a flux increase occurs due to the fuel change in one region, a nonregional regulating system would compensate for this and maintain steady power by lowering the flux in another region to keep the average flux across the core constant. This would set up a xenon oscillation in the second region exactly out of phase with that in the first region. When the first region becomes supercritical the second region becomes subcritical and when the first region becomes subcritical the second region becomes supercritical.

The period of the flux and power oscillations in any one region is about 20 to 30 hours. Such oscillations of power are most undesirable and so, in large reactors, different regions of the core must have some independence of control. Neutron absorber rods are better suited for such independent control. As at Douglas Point, regional absorber rods are regulated by an independent control mechanism fed by independent local flux detectors. At the same time, moderator level control is used for general reactor regulation.

ASSIGNMENT

1. (a) What basic condition helps a reactor to be stable to temperature changes?

1. (b) What factors may cause high frequency power instabilities?

   (c) How can a reactor be inherently stable, even though the condition in (a) is not satisfied, provided that it has a negative power or fuel temperature coefficient?

2. (a) Why are xenon oscillations more likely to occur in a large reactor than in a small one?

   (b) Describe how such xenon oscillations occur in such an inherently unstable reactor.

   (c) How are such xenon oscillations prevented?

A. Williams

Nuclear Theory - Course 127

SAFETY CONSIDERATIONS

No combination of engineering know-how and protective devices will guarantee an accident-free station. However, first emphasis must still be placed on the prevention of accidents and the reduction of probability of hazards arising. In the event of failure of the measures taken to prevent accidents occurring, further consideration must be given to measures which can be taken to reduce the consequences of such accidents.

The description of the measures which can be taken may well be a summary of the various topics already discussed in the Nuclear Theory Course and therefore this final lesson may, to some extent, summarize aspects of nuclear theory which are pertinent to plant safety.

Classification of Hazards

Hazards to personnel and equipment have to be considered and these may have little or nothing to do with reactor regulation or protection. For convenience, three categories of hazards are defined as follows:

(a) Normal Hazards - These are hazards which are always present because of the character of a nuclear-electric generating station or because of conceivable errors in operation. Exposure to radiation is a typical example of this type of hazard.

(b) Minor Emergencies - Such emergencies are caused by malfunctioning of controls or by unexpected failures of components. An example of such a minor emergency is the drift that may occur in the set point of a tripping device. Another example is that of low flow in one reactor channel causing boiling which in turn causes a power transient because of the void coefficient.

(c) Major Emergencies - These are produced by circumstances beyond the control of anyone associated with the design, construction or operation of the station. They may be defined as a nuclear incident which could not be or was not stopped by the regulating or protective devices. Typical consequences of such emergencies would be the release of fission products inside and outside the station, or even the melting of the reactor core.

Avoidance of Normal Hazards

The occurrence of normal hazards can be reduced or eliminated by applying the following principles during the design, construction or operation of the station:

(a) Shielding around radioactive sources or areas should be such as to maintain personnel exposure to well below the allowable levels. An exposure bank or reserve can then be accumulated for maintenance or emergencies.

(b) Fixed area monitors should be installed or portable monitors carried which alarm when there is an unexpected increase in the radiation field.

(c) Door interlocks should be provided which prevent access to certain areas during periods of excessive radiation levels and which also prevent such excessive radiation levels when access to such areas are possible.

(d) Contamination of personnel must be kept to a minimum. Such factors as personnel monitoring, protective clothing, showers, and change room and laundry facilities, are important aids in this direction.

(e) Contamination levels throughout the station should be kept as low as possible. Swipe checks should be taken at regular intervals to ensure this. Regular cleaning and decontamination is required with spills cleaned immediately. Cross contamination is avoided by using a zoning system.

(f) Gaseous and liquid effluents must be carefully monitored and controlled.

(g) Adequate facilities should be available for transportation and disposal of spent fuel and radioactive waste.

(h) All operating and maintenance personnel should receive thorough training in all aspects of nuclear-electric generating station operations. Radiation Protection training should be included and all training should be slanted so as to develop a "safety attitude" in all station personnel.

(i) It is desirable that personnel, who will eventually operate the station, should participate in the commissioning and testing phase and that at least some should be closely associated with the design of the station.

Reducing the Probability of Minor Emergencies

The probabilities of minor emergencies occurring are, of course, reduced when normal engineering safety considerations

are applied. Thus, only equipment and material should be used, the performances of which under various operational conditions are either accurately known or can be determined. There should obviously be strict adherence to applicable codes and regulations and inspection and testing should be a scheduled feature of procurement, construction and operation. In addition, the following principles should also be applied:

(a) Reactivity control mechanisms must be such as to prevent unsafe reactivity rates of increase or unsafe temperature increase rates.

(b) It is desirable that the reactor has inherently safe thermal nuclear characteristics, ie, the net temperature coefficient of reactivity is negative and the reactor at least partially self-regulating.

(c) Void coefficients should not be excessively positive or negative. Excessive positive coefficients cause large transient power surges during the void formation. Excessive negative coefficients cause the regulating system to make violent corrective adjustments when the void occurs, which result in a power surge when the void collapses or fills up.

(d) It is desirable that regulation should be on two channels to minimize the effects of instrument failures. It is an added advantage to be able to inspect and test regulating equipment regularly during operation. This makes it desirable to have triplicated regulating channels - adequate regulation being achieved on any two of the three channels.

(e) All operations of a reactor should be carried out according to a prearranged sequence. Interlock circuits are required to ensure that such a sequence is followed and that an improper sequence cannot be followed.

No operating sequence should be initiated unless sufficient indication is always available of the condition of the reactor. If necessary, additional instrumentation must be incorporated for the initial approach to critical.

(f) Audible annunciations and visual indications are necessary to show that unsatisfactory conditions exist or are developing.

(g) A protective system is required to trip the reactor when certain critical variables exceed or become less than a predetermined limit. Such variables (eg, neutron power, reactor period, high temperature, low pressure, etc) may exceed or become less than the predetermined level because of failure of the regulating system, lack of inherent safe

characteristics or failure of components in any other system. The protective system should cause a rapid decrease in reactivity when such a variable causes a trip.

(h) Such a protective system should fail safe, ie, if an instrument or component in the system should fail, a reactor trip should be initiated. In order to prevent a single component failure from causing a reactor trip, and in order to enable regular testing to be carried out, three protective channels are an advantage, adequate protection being obtained when two out of three protective channels initiate a trip.

(i) Audible annunciation and visual indicators should show which variable caused a trip so that adequate investigation can be carried out following a trip.

Reducing the Consequences of Major Emergencies

The types of accidents which are likely to cause a major emergency are:

(a) Loss of coolant which might result from a failure or break in the heat transport system with a consequent appreciable leak of coolant. This would likely set up voids in the core with a resulting increase in reactivity and, if the loss is severe, the fuel elements and coolant channels would no longer be cooled and might melt.

(b) Loss of control on startup or on-power, due to the failure of the regulating system to operate in such a manner as to prevent a power excursion, and in addition the protective system fails to trip the reactor. It is likely that, under such circumstances, the coolant would boil causing steam-blanketing of the fuel. This in turn leads to sheath failure followed by melting or disintegration of fuel elements. If the fuel and coolant enter the moderator, enough heat might be provided to boil the moderator sufficiently to rupture the moderator system.

Such a loss of control can also occur from the shutdown state as distinct from during a startup. A reactor is in its most dangerous condition when it is shut down for the reasons listed in a previous lesson.

(c) Loss of cooling which does not involve the immediate disappearance of fluid from the heat transport system. It merely indicates a decrease or cessation of normal heat transport flow through any or all of the fuel channels, due to pump failure, channel constriction or inadvertent closure of isolating valves. Such a loss of cooling would tend to result in the formation of a vapour blanket over the fuel which, in turn, would cause excessive fuel temperatures.

(d) Loss of boiler water resulting from a break in the steam main or from feedwater pump failure. Such an incident would result in an immediate large drop in steam pressure. This would in turn cause a demand for increased reactor power, which should result in an overpower trip. If the trip fails to operate, coolant boiling would probably result which would again cause excessive fuel temperatures.

Measures which are taken to reduce the consequences of a major emergency include those already taken during the design and construction of the station and those which will be taken if and when the emergency arises. Possible measures which may be taken are, briefly, as follows:

(a) Use of Containment Systems

Containment starts at the fuel itself. Uranium oxide is able to contain a high percentage of the fission products formed in it because of slow diffusion rates through the high density oxide. The fuel is also sheathed and the sheath acts as a fission product container. On sheath failure, fission products are released into the heat transport system and the heat transport system acts as a containment system and should be leak-tight.

However, if a failure or break occurs in the heat transport system, additional containment may be required to prevent the spread of fission products, especially if the fuel melts or disintegrates as a result of loss of cooling. The reactor and heat transport system are located within a sealed containment shell designed to withstand any pressure buildup which may occur as a result of the accident. Since such a shell has to house the reactor boiler plant and the associated shielding and material-handling facilities, it would of necessity be very large.

The shell would have to be thick enough to withstand pressure increases of the order of 40 psia or more, and also to withstand the thermal stresses resulting from the heat released. It would be impossible to reduce leakage from such a vessel to zero under such pressures and therefore the station would still have to be located in an exclusion area large enough to allow radioactive fallout within this area. In general, the erection of such a vessel increases the plant cost by 3% to 5%.

(b) Use of Pressure Relief Systems

In this type of system the building itself may be used to contain the radioactive materials. However, to avoid the

effect of the initial pressure increase on the structure, the initial blast resulting from the explosion is allowed to escape through a relief duct into the atmosphere.

Such a relief duct, as used in NPD, is shown in Fig. 1. This relief of pressure reduces the pressure buildup from the explosion and the structural thicknesses required to withstand such a pressure. It is assumed that the initial escape of steam and air only contains tritium, since fuel failure will only occur subsequent to the loss of heat transport fluid. Immediately following the initial blast, a door closes in the relief duct, preventing escape of fission products released after fuel failure.

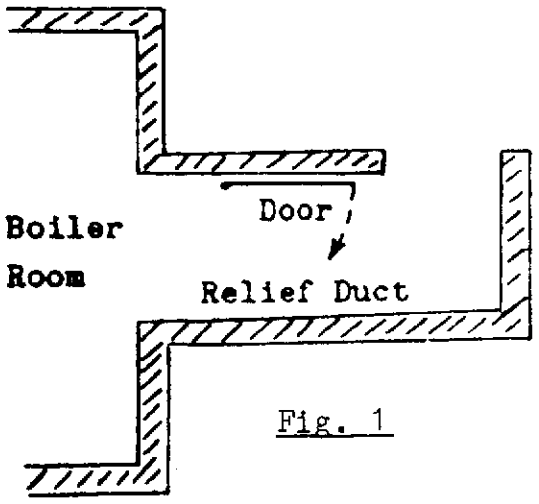


Fig. 1

Pressure relief may be used, as in Pickering G.S., as a means of reducing the pressure buildup even though a containment shell is used. Such a pressure relief would reduce the thickness of the containment shell required. The type of arrangement proposed is as shown in Fig. 2. Each reactor and associated heat transport system will be housed in a 4-ft thick cylindrical concrete containment building. The reactor buildings will connect, through ducts, to a relief vessel. This pressure relief vessel will be kept under a continuous vacuum. Any pressure increase in a reactor building will thus be relieved continuously into the relief vessel.

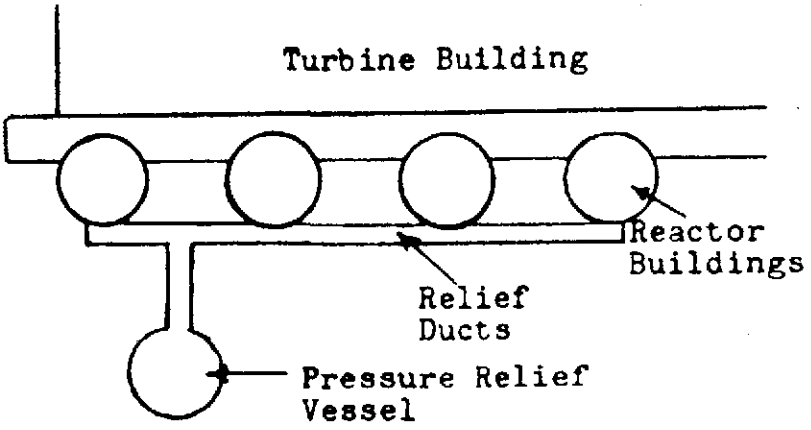


Fig. 2

(c) Use of Pressure Suppression

Since most of the accidents that can be contemplated result in a violent release of steam containing radioactive material,

containment of the radioactive material is effected if all the steam can be condensed. Such pressure suppression can be obtained by allowing the pressure buildup to initiate a dousing system which spray-cools the steam. The dousing system may have to be used with a pressure relief system or a containment shell. However, the shell would be thinner and less expensive. It was estimated that, without dousing, a containment system for NPD would have to withstand a 40-psia increase in pressure with 10^6 Btu of heat released. In one Pickering unit, around 2.4×10^8 Btu of heat would have to be dissipated during the type of accident envisaged; yet at Douglas Point, with dousing but no pressure relief, the maximum internal pressure estimated in the containment vessel is only 6 psi. Consequently, the containment vessel walls can be made of 4-ft thick concrete with a corresponding thickness of steel in the dome. The leakage rate out of this enclosure can be kept down to 0.1% of the total volume per hour.

An alternative arrangement is that shown in Fig. 3. The reactor and heat transport system are contained in a dry well filled with air or carbon dioxide. The dry well vents to a water pool enclosed in a vapour-tight container. As pressure increases in the dry well, air and steam vent into the pool, where the steam is condensed.

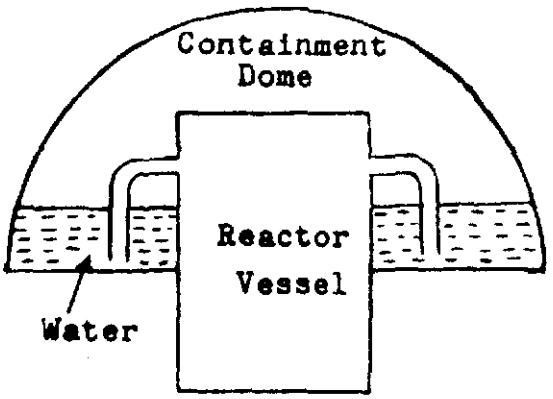


Fig. 3

(d) Use of Injection Systems

Fuel failure and resulting fission product release inevitably results from lack of heat removal. If the fuel can be prevented from rupturing or melting, then the core remains intact and the fission products are contained in the fuel. A system can therefore be designed so that additional fluid can be pumped into the system in the event of excessive leakage. In D_2O systems the initial injection would be of D_2O but should this be used up then to save the core, an auxiliary H_2O supply would be available as an additional measure.

(e) Use of Suitable Procedures

Despite the measures described above for reducing the consequences of a major emergency, it is still very necessary to establish a formal guide to cover the procedures that should be followed in the event of such an emergency. Such a guide, in the form of an operating manual known as Radiation Incidents and Emergencies, would indicate the general method of handling such situations and serve as a basis for training the station personnel and for carrying out emergency drills. Such a manual would:

(1) Ensure adequate local arrangements and make provisions for outside assistance.

(2) Indicate how to deal with the public and what post-incident evaluations are required.

(3) Give guidance on the classification of the emergency.

(4) Give guidance on the assumption of command, the setting up of command points, the setting up of sectors and the arrangements for surveying such sectors, the general action to be taken and the method of assembly of personnel.

Such a guide is discussed fully under Radiation Protection Procedures.

ASSIGNMENT

1. Briefly define the three categories of hazards.

2. As briefly as possible, list three factors, incorporated into the design of a station, which help to avoid normal hazards.

3. What three operational measures can be taken to minimize normal hazards?

4. List three "built-in" characteristics of a reactor or its associated regulating system that help to reduce the probability of minor emergencies occurring.

5. Why is it desirable to have triplicated regulating and protective systems?

6. (a) What basic principles should be followed, in reactor operation, to reduce the probability of a minor emergency?

6. (b) How is it possible to ensure that such principles are followed?

7. List the four likely causes of a major emergency.

8. (a) What are the various stages of containment of fission products that are usually present in a nuclear-electric generating station?

   (b) What additional method of containment could be used and what specifications would such a containment system have to meet?

9. Briefly describe three other design measures that could be used to reduce the consequences of a major emergency.

A. Williams